XII PHYSICS

Chapter #
11, 12, 13, 14, 17

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XII PHYSICS

NOTES

CHAPTER # **11**

HEAT

HEAT:

"Heat is defined as the energy transferred from one body to the other because of the temperature difference between them by conduction, convection or radiation."

OR

"Heat is defined as the total kinetic energy of the molecules of a body."

Unit of Heat:

a) CGS unit of heat is calorie. 1 calorie is the amount of heat required to raise the temperature of 1 g of pure water through 1 ^{o}C (from 14.5 ^{o}C to 15.5 ^{o}C). It dietician calorie is [1 Cal. = 1000 cal. = 1 kcal.]
b) MKS (or SI) unit of heat is joule. [1J = 1N. 1m]. 1 joule is the work done by a force of 1N when its point of application moves through 1m in the direction of force. [1 cal. = 4.186 J]
c) The British unit of heat is called British thermal unit (Btu).

TEMPERATURE:

The degree of hotness or coldness of a body is known as temperature or more specifically temperature may be defined as "average kinetic energy (of translation) of the molecules of a body."

THERMAL EQUILIBRIUM:

"When two bodies at different temperature are brought in contact then heat flows from hot body to cold one. After some time the temperature of both the bodies becomes same then they are said to be in thermal equilibrium."

MEASUREMENT OF TEMPERATURE:

The instrument used to measure temperature is called thermometer. The determination of temperature requires (i) Thermometric property of the substance that changes uniformly with temperature and (ii) temperature scale that comprises of two reference points and they must be reproduceable.

TEMPERATURE SCALES

1) CELSIUS OR CENTIGRADE SCALE:

In Celsius scale melting of ice (0^{o}C) is taken as lower fixed point (l.f.p.) and boiling of water (100^{o}C) is taken as upper fixed point (u.f.p.) and both temperatures are taken at standard pressure (760mmHg). This interval is divided into 100 parts. Each part is 1^{o}C.

2) FAHRENHEIT SCALE:

In this scale the lower fixed point (l.f.p) is taken 32^{o}F and (u.f.p) is taken 212^{o}F the scale is divided into 180 equal parts. Each part is 1^{o}F.

3) KELVIN SCALE:

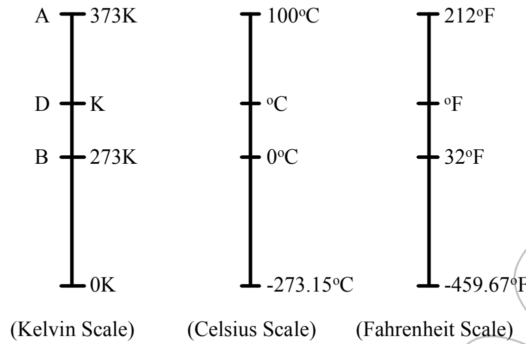
The lowest possible temperature to achieve is -273.15^{o}C (called 'Absolute zero'). This is taken as 'O' Kelvin on Kelvin scale. 1 degree of Celsius and Kelvin scale is equal therefore

O K = -273 ^{o}C

273 K = 0 ^{o}C

373 K = 100 ^{o}C

INTERCONVERSION OF TEMPERATURE SCALES:



From figure 11.1

$$\frac{DB}{AB} = \frac{{}^{o}C - 0}{100 - 0} = \frac{{}^{o}F - 32}{212 - 32}$$

$$\frac{{}^{o}C}{100} = \frac{{}^{o}F - 32}{180}$$

$${}^{o}C = \frac{100}{180}({}^{o}F - 32)$$

$$\boxed{{}^{o}C = \frac{5}{9}({}^{o}F - 32)}$$

This formula is used for interconversion of ^{o}C & ^{o}F.

Besides

$$\boxed{K = {}^{o}C + 273}$$

This formula is used for interconversion of K & ^{o}C.

THERMAL EXPANSION:

When things are heated, they undergo a change in size. The increase in size due to heat is called thermal expansion.

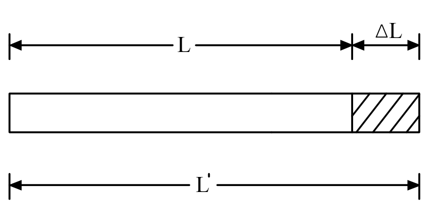
Infact due to heat the body temperature rises and the molecules of substance vibrate more energetically and their amplitude increases and because the average distance between molecules increases therefore the size of object increases.

There are two kinds of thermal expansion.

1) Linear Expansion
2) Cubic or Volume Expansion

1) LINEAR EXPANSION:

"Thermal expansion in the length of an object is called linear expansion. Linear expansion occurs only in the solids of linear shape."



Consider a metallic rod of original length ‘L’ and initial temperature ‘T_1’. When its temperature is raised to ‘T_2’ then its final length becomes ‘L´’ with a change of length ‘ΔL’.

Experimentally it is known that change in length ‘ΔL’ directly varies with original length ‘L’ and change in temperature ‘ΔT’ therefore

$$\Delta L \; \alpha \; L$$

& $$\Delta L \; \alpha \; \Delta T$$

combining these factors

$$\Delta L \; \alpha \; L \, \Delta T \qquad \rightarrow \text{(i)}$$

or $$\Delta L = \alpha \, L \, \Delta L$$

Where ‘α’ is constant of proportionality and known as ‘co-efficient of linear expansion’.

Furthermore

$$\boxed{\alpha = \frac{\Delta L}{L \, \Delta T}}$$

‘α’ may be defined as “the change in length per unit original length per degree Kelvin rise of temperature.”

The unit of ‘α’ is K^{-1} or $^{o}C^{-1}$.

From equation (i)

$$\Delta L = \alpha \, L \, \Delta T$$

or $$L' - L = \alpha \, L \, \Delta T$$

$$L' = L + \alpha \, L \, \Delta T$$

or $$L' = L \, (1 + \alpha \, \Delta T)$$

or $$\boxed{L' = L \, \{1 + \alpha \, (T_2 - T_1)\}}$$

This expression may be used for determine of find length of object.

2) <u>CUBIC OR VOLUME EXPANSION:</u>

“Thermal expansion in which the volume of a body expands is called Cubic or Volume expansion. All physical states of matter (solid, liquid & gas) show this property of expansion in volume on heating.”

Consider a body having length ‘L’, width ‘W’ & height ‘H’ at temperature ‘T_1’. After heating upto temperature ‘T_2’ its dimensions become L´, W´ & H´ respectively, therefore

Original volume = V = L x W x H
Final volume V´ = L´ x W´ x H´
Change in temperature = $\Delta T = T_2 - T_1$

Experimentally it is known that change in volume ‘ΔV’ is directly proportional to the original volume & change in temperature.

$$\Delta V \; \alpha \; V$$

& $$\Delta V \; \alpha \; \Delta T$$

combining these factors

$$\Delta V \propto V \Delta T$$

or $$\Delta V = \beta V \Delta T \rightarrow \text{(i)}$$

Where 'β' is constant of proportionality and it is known as 'co-efficient of volume expansion'.

Furthermore

$$\boxed{\beta = \frac{\Delta V}{V \Delta T}}$$

'β' may be defined as "change in volume per unit original volume per unit (Kelvin) rise of temperature."

Its unit is K^{-1} or $^{o}C^{-1}$

From equation (i)

$$\Delta V = \beta V \Delta T$$

or $$V' - V = \beta V \Delta T$$

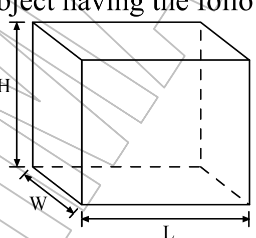
or $$V' = V + V \beta \Delta T$$

or $$\boxed{V' = V \{1 + \beta (T_2 - T_1)\}}$$

This expression may be used for the determination of final volume.

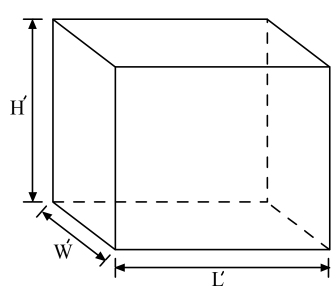
<u>To Prove β = 3 α</u>

Consider a cubical or rectangular object having the following dimensions at temperature T_1.



Length = L
Width = W
Height = H
Original volume = V = L x W x H

When its temperature is raised to T_2 then its dimensions become L′, W′ & H′ respectively.



$$V' = L' \times W' \times H'$$

But $$L' = L (1 + \alpha \Delta T)$$

$$W' = W (1 + \alpha \Delta T)$$

$$H' = H\,(1 + \alpha\,\Delta T)$$

$$V' = L\,(1 + \alpha\,\Delta T).W\,(1 + \alpha\,\Delta T).H\,(1 + \alpha\,\Delta T)$$

or $$V' = LWH\,(1 + 3\,\alpha\,\Delta T)^3$$

or $$V' = V\,(1 + 3(1)^2\alpha\Delta T + 3(1)\alpha^2\Delta T^2 + \alpha^3\Delta T^3)$$

Neglecting $3\alpha^2\Delta T^2$ & $\alpha^3\Delta T^3$ as they are so small

$$V' = V\,(1 + 3\,\alpha\,\Delta T)$$

or $$V' = V + 3\,V\,\alpha\,\Delta T$$

or $$V' - V = 3\,V\,\alpha\,\Delta T$$

or $$\Delta V = V\,\Delta T\,3\,\alpha$$

or $$\frac{\Delta V}{V\,\Delta T} = 3\alpha$$

But $$\frac{\Delta V}{V\,\Delta T} = \beta$$

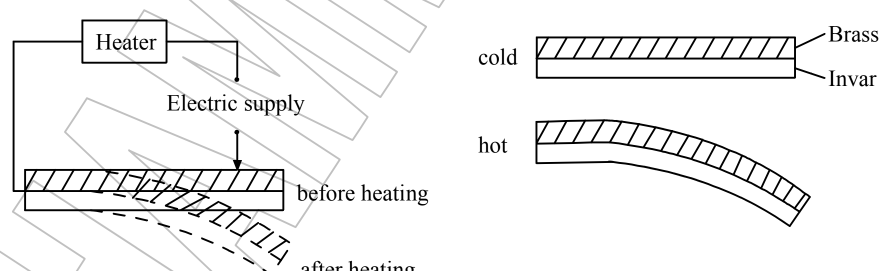
$$\boxed{B = 3\,\alpha}$$ Proved

APPLICATIONS OF THERMAL EXPANSION:

The phenomenon of thermal expansion may be utilized in different purposes. Some typical examples are given below:

1) BIMETALLIC THERMOSTAT:

Thermostat is a device which is used to control temperatures in different instruments and objects like refrigerators, ovens, laundry iron, hot water storage tanks, etc.



BIMETALLIC THERMOSTAT

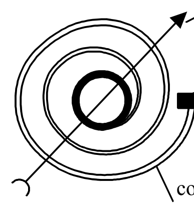
PRINCIPLE:

Every material has its own coefficient of linear expansion which means that the rate of expansion in different materials is different. Therefore using this fact we make a bimetallic strip by joining one metal strip onto the other. When temperature rises the metallic strip bend because the metal with high rate of expansion expands rapidly and it is always on the outside curve of the both strips.

WORKING:

In an electric heating circuit, the bimetallic strip works as an electric contact breaker. When temperature rises, the bimetallic strip bends and the contact is broken. The current stops to flow. When the temperature falls, the strip comes back into contact again and the current begins to flow again and in this way a particular temperature is maintained.

2) BIMETALLIC THERMOMETER:



PRINCIPLE:

A bimetallic thermometer works on the principle of differential expansion. It is used to read temperature of hot regions.

WORKING:

A thin bimetallic spiral is taken, whose one end is fixed. The other end is attached to a spindle of a pointer which moves over a scale. As the temperature rises, the spiral tends to bend (in clockwise direction). This causes the pointer to move. The scale helps to read the temperature.

They are used in automobiles, oven and for air thermometers.

GAS LAWS:

Gas laws are such equations or relations that relate parameters of a gas such as mass, volume, Temperature, Pressure, etc.

BOYLE'S LAW:

Statement:

This law states that

"The volume of a given mass of a gas is inversely proportional to the pressure, provided temperature is kept constant."

Robert Boyle presented the above law is 1660.

Mathematical Representation:

$$\text{Volume} \ \alpha \ \frac{1}{\text{Pressure}}$$

i.e. $V \ \alpha \ \frac{1}{P}$ (at constant temperature)

or $V = K \ \frac{1}{P}$

Here 'K' is the constant of proportionality.

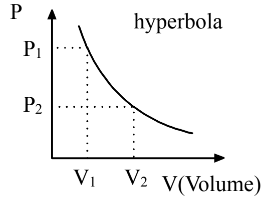
or $PV = K$

Therefore according to Boyle's law if temperature is kept constant then the product of pressure and volume of a given mass of a gas always remains constant.

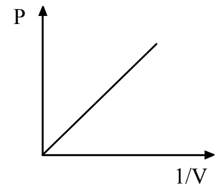
For two different states

$$P_1V_1 = P_2V_2$$

If a graph between P & V is drawn at constant temperature then it is a curve, called hyberbola.



OR



It is observed practically that the product PV in proportional to the mass of gas.

$$\frac{PV}{m} = \text{constant}$$

or $$\frac{P_1V_1}{m_1} = \frac{P_2V_2}{m_2}$$

If ρ_1 & ρ_2 refer to the densities of two samples of a gas then,

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$$

<u>CHARLE'S LAW:</u>

<u>Statement:</u>

This law states that

"The volume of a given mass of a gas is directly proportional to the absolute temperature provided pressure is kept constant."

This law was given by Jacques A.C. charles in 1787.

<u>Mathematical Representation:</u>

Volume α Temperature (absolute)

or $V \alpha T$ (at constant Pressure)

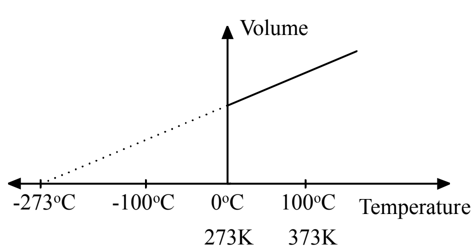
or $V = KT$

Where K is the constant of proportionality

$$\frac{V}{T} = K$$

For two different states

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



The graph between volume and temperature is a straight line. If the graph is extra plotted backward, it cuts the temperature axis at -273 °C. This shows that the volume of the gas is zero. Hence it is concluded that -273 °C is the lowest conceivable temperature. Kelvin selected this temperature (-273 °C) as the zero, called "Absolute zero" (0 A or 0 K). A temperature scale on which -273 °C is taken as zero is called "Absolute Scale" or "Kelvin Scale" or Thermodynamic Temperature Scale.

$$0\ ^oC = 273.15\ K$$

GENERAL GAS LAW:

General gas law or ideal gas law is such mathematical relationship which relates the parameters such as pressure, volume, temperature and amount of matter of a given sample of a gas and it describes the behaviour of a gas. It is also known as 'equation of States'.

Derivation:

Consider a given mass of a gas occupying a space (volume) V_1, Pressure P_1 at temperature T_1. Suppose its pressure is changed to P_2 keeping temperature constant and its new volume becomes V_x.

According to Boyle's law

$$P_1V_1 = P_2V_x$$

or $$V_x = \frac{P_1V_1}{P_2} \rightarrow \text{(i)}$$

Now if the temperature is changed to T_2, keeping the temperature constant then the new volume is V_2.

According to Charle's law

$$\frac{V_x}{T_1} = \frac{V_2}{T_2}$$

or $$V_x = \frac{V_2T_1}{T_2} \rightarrow \text{(ii)}$$

Comparing equation (i) & (ii)

$$\frac{P_1V_1}{P_2} = \frac{V_2T_1}{T_2}$$

or $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$ or $$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

or $$\frac{PV}{T} = R_m$$

Where 'R_m' is the constant of proportionality and it has different values for different samples of gases. It mainly depends on the (i) mass of the gas (ii) nature of the gas (iii) units of P & V and T. Therefore it is different for different samples of gas since they differ in mass i.e. no. of molecules.

According to Avogadro, one mole of any substance always contains 6.02×10^{23} atoms or molecules. Thus by selecting one mole of any gas, we refer to the number of molecules independent of the gas.

Therefore to generalize gas constant we take the mass of gas in moles and S.I. units of P, V & T. Then the gas constant is called 'universal gas constant' or the 'molar gas constant' denoted by R. It is independent of the gas in a sample. In S.I. units its value is 8.31 $J.mole^{-1}.K^{-1}$.

For one mole gas constant is R

For two moles gas constant is 2R
For three moles gas constant is 3R

Hence for 'n' moles it should be nR

$$\frac{PV}{T} = nR$$

or $$\boxed{PV = nRT}$$

This equation is known as "General Gas Equation".

UNIT OF 'R'

Since $PV = nRT$

& $R = \frac{PV}{nT}$

In M.K.S system $R = \frac{N}{m^2} . m^3 \Big/ \text{mole. K}$

or $R = N.m.mole^{-1}.K^{-1}$: J(Joule) = N.m

or $R = J.mole^{-1}.K^{-1}$

VALUE OF 'R'

At S.T.P. one mole of a gas has the following data.

No. of moles $n = 1$ mole

Volume $V = 22.41\ dm^3$ or $0.02241\ m^3$

Pressure $P = 1$ atm or $1.01325 \times 10^5\ N/m^2$

Temperature $T = 0\ ^oC$ or 273 K

Substituting all values in general gas equation

$$R = \frac{PV}{nT} = \frac{1.01325 \times 10^5\ N/m^2 \times 0.02241\ m^3}{1\ \text{mole} \times 273\ K}$$

$$R = 8.313\ N.m.mole^{-1}.K^{-1}$$

or $$\boxed{R = 8.313\ J.mole^{-1}.K^{-1}}$$

IDEAL GAS:

A gas that obeys all gas laws is called ideal gas.

REAL GAS:

A gas that obeys gas laws at high temperature and low pressure only.

KINETIC MOLECULAR THEORY OF GASES:

To explain the macroscopic behaviour or properties of a gas on microscopic basis the kinetic model of gas or kinetic molecular theory was proposed. For the very first time it was proposed by Hermann in 1738. Later it was improved by many other scientists like Boltzman, Maxwell, Bernoulli, Gibbs, Helmholtz etc.

This theory consists of the following assumptions.

ASSUMPTIONS:

1) All the gases comprise of very very tiny particles that are 'point masses' and are known as molecules. These molecules have large distance among them as compared to their size.
2) The molecules of a gas are always in continuous random motion. But this motion is always through a straight line.
3) During their random motion, molecules collide with each other and the walls of container. These collisions are elastic.
4) The time consumed in collision is negligible as compared to the time required to cover distance between two molecules.
5) Molecules of a gas do not have any mutual attraction.
6) They are free to move in all the directions (x, y, z axes).
7) Laws of Mechanics are applicable to the motion of molecules.

INTERPRETATION OF PRESSURE OF A GAS:

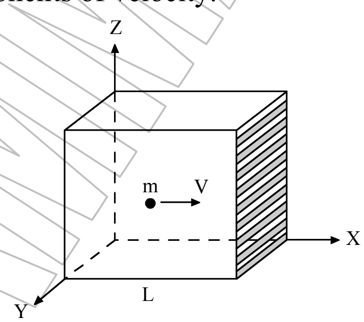
Pressure of a gas can be interpretation using kinetic theory of gases as following:
First of all we should know that pressure is defined as the ratio of Force & Area

i.e. $$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

or $$P = \frac{F}{A}$$

Where Force F = Rate of change in momentum of molecules of a gas in case of pressure of a gas.
Now suppose a cubical container contains some amount of a gas such that;

Volume of container = $V = L \times L \times L = L^3$
Mass of a molecule = m
Total number of molecules = N
Velocity of a molecule = V
Where V_x, V_y & V_z are components of velocity.



Now consider a single molecule moving along X-axis and it strikes the shaded face of the container elastically. It rebounces or reversed that is 'V_x' is reversed and V_y & V_z remains same.

Initial momentum = mV_x [momentum = mass x velocity]

And after collision

Final momentum = $-mV_x$ [since V_x reversed]

Change in momentum = $mV_x - (-mV_x)$

$= mV_x + mV_x$

$$= 2mV_x$$

or $$\text{Rate of change of momentum} = \frac{2mV_x}{t} \rightarrow (A)$$

We know that

$$\text{Time of impact} = \frac{\text{distance traveled}}{\text{Speed}}$$

or $$t = \frac{2L}{V_x}$$

Putting in equation (A)

$$\text{Rate of change of momentum} = \frac{2mV_x}{2L/V_x}$$

$$= \frac{mV_x^2}{L}$$

But in beginning we have seen that rate of change of momentum is equal to force.

$$f \text{ (force)} = \frac{mV_x^2}{L}$$

Now if we talk about the net force of all the molecules then

$$\text{Net Force } F = f_1 + f_2 + f_3 + \ldots\ldots\ldots + f_n$$

or $$F = \frac{m}{L}V_{1x}^2 + \frac{m}{L}V_{2x}^2 + \frac{m}{L}V_{3x}^2 + \ldots\ldots\ldots + \frac{m}{L}V_{nx}^2$$

or $$F = \frac{m}{L}(V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \ldots\ldots\ldots + V_{nx}^2)$$

or $$F = \frac{m}{L}\sum_{i=1}^{i=n} V_x^2 \rightarrow (B)$$

We also know that

$$P = \frac{F}{A}$$

But $A = L^2 = L \times L$ & $F = \frac{m}{L}\sum_{i=1}^{i=n} V_x^2$

Putting these values

$$P = \frac{\frac{m}{L}\sum_{i=1}^{i=n} V_x^2}{L^2}$$

or $$P = \frac{m}{L^3}\sum_{i=1}^{i=n} V_x^2 \rightarrow (C)$$

Let the number of molecules per unit volume = n_v

Then $$n_v = \frac{N}{V}$$

or $$n_v = \frac{N}{L^3} \qquad [\,V = L^3\,]$$

or $$L^3 = \frac{N}{n_v}$$

Putting in equation (C) we get

$$P = \frac{m}{N/n_v} \sum_{i=1}^{i=n} V_x^2$$

or $$P = \frac{mn_v}{N} \sum_{i=1}^{i=n} V_x^2 \qquad \rightarrow (D)$$

We know that (ℓ) density = $\frac{\text{Mass}}{\text{Volume}} = \frac{mN}{V} = m\,\frac{N}{V} = mn_v$

Putting in (D)

$$P = \frac{\ell}{N} \sum_{i=1}^{i=n} V_x^2$$

or $$P = \ell\left(\frac{\sum_{i=1}^{i=n} V_x^2}{N}\right) \qquad \rightarrow (E)$$

But mean square velocity, $\overline{V_x^2} = \frac{V_{1x}^2 + V_{2x}^2 + V_{3x}^2 + \ldots\ldots\ldots + V_{nx}^2}{N}$

Where $N = N_1 + N_2 + N_3 + \ldots\ldots + N_n$ (total no. of molecules).

Equation (E) becomes

$$P = \ell\,\overline{V_x}^2 \qquad \rightarrow (F)$$

Now

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

or $$V^2 = V_x^2 + V_y^2 + V_z^2$$

or $$\overline{V}^2 = \overline{V_x}^2 + \overline{V_y}^2 + \overline{V_z}^2$$

Since all the components of velocity are alike

$$\overline{V_x}^2 = \overline{V_y}^2 = \overline{V_z}^2$$

or $$\overline{V}^2 = \overline{V_x}^2 + \overline{V_x}^2 + \overline{V_x}^2$$

or $$\overline{V}^2 = 3\overline{V_x}^2$$

or $\overline{V_x^2} = \frac{1}{3} \overline{V^2}$ → (G)

Putting $\overline{V_x^2}$ in equation (F)

$$\boxed{P = \frac{1}{3} \ell \overline{V^2}}$$ Proved

INTERPRETATION OF TEMPERATURE:

According to Kinetic Molecular theory

$$P = \frac{1}{3} \ell \overline{V^2}$$

or $P = \frac{1}{3} mn_v \overline{V^2}$ [$\rho = mn_v$]

or $P = \frac{1}{3} m \frac{N}{V} \overline{V^2}$ [$n_v = N/V$]

or $PV = \frac{1}{3} mN\overline{V^2}$ → (i)

we know that

$PV = nRT$ → (ii)

Comparing equation (i) & (ii)

$$\frac{1}{3} mN\overline{V^2} = nRT$$

or $\frac{1}{3} mN\overline{V^2} = \frac{N}{N_A} RT$ [$n = \frac{N}{N_A} = \frac{\text{No. of molecules}}{\text{Avogadro's No.}}$]

or $\frac{1}{3} m\overline{V^2} = \left(\frac{R}{N_A}\right) T$

Here $R/N_A = K$, where ‚K' is Boltzman's constant and it is defined as universal gas constant per molecule.

or $\frac{1}{3} m\overline{V^2} = KT$

or $m\overline{V^2} = 3KT$

or $\frac{1}{2} m\overline{V^2} = \frac{3}{2} KT$

or $\boxed{(K.E)_{Avg} = \frac{3}{2} KT}$

or $(K.E)_{Avg} = (\text{constant})\ T$ [$3/2 . K = \text{constant}$]

or $\boxed{(K.E)_{Avg}\ \alpha\ T}$

It shows that the absolute temperature is directly proportional to the average translational kinetic energy of the molecules.

ROOT MEAN SQUARE VELOCITY OF MOLECULE (V_{rms}):

We know that

$$P = \frac{1}{3} \ell \overline{V^2}$$

or $$\overline{V^2} = \frac{3P}{\ell}$$

or $$\sqrt{\overline{V^2}} = \sqrt{3P/\ell}$$

or $$\boxed{V_{rms} = \sqrt{\frac{3P}{\ell}}}$$

we also know that

$$(K.E.)_{Avg} = \frac{3}{2} KT$$

or $$\frac{1}{2} m\overline{V^2} = \frac{3}{2} KT$$

or $$\overline{V^2} = \frac{3}{2} KT.\frac{2}{m}$$

or $$\overline{V^2} = 3KT$$

or $$\sqrt{\overline{V^2}} = \sqrt{\frac{3KT}{m}}$$

or $$\boxed{V_{rms} = \sqrt{\frac{3KT}{m}}}$$

or $$V_{rms} = \sqrt{\frac{3KT}{m}}$$

or $$V_{rms} = \sqrt{\frac{3.(R/N_A).T}{m}} \qquad [\ K = R/N_A\ ]$$

or $$V_{rms} = \sqrt{\frac{3RT}{mN_A}} \qquad [\ mN_A = M\ ]$$

or $$\boxed{V_{rms} = \sqrt{\frac{3RT}{M}}}$$

Where 'M' is molecular mass of gas in kg/mole.

KINETIC INTERPRETATION OF GAS LAWS

DEDUCTION OF BOYLE'S LAW:

According to kinetic molecular theory

$$P = \frac{1}{3} \ell \overline{V^2}$$

or $$P = \frac{1}{3} mn_v \overline{V^2} \qquad [\ell = mn_v]$$

or $$P = \frac{1}{3} m \frac{N}{V} \overline{V^2} \qquad [n_v = N/V]$$

or $$PV = \frac{1}{3} mN\overline{V^2}$$

or $$PV = \frac{2}{3} N \frac{1}{2} m\overline{V^2} \qquad [\tfrac{1}{2} mV^2 = (K.E.)_{Avg} = 3/2 . KT]$$

or $$PV = \frac{2}{3} N \frac{3}{2} KT$$

or $$PV = NKT$$

Now if 'T' is kept constant then 'NKT' is a constant and

$$\boxed{PV = \text{Constant}} \qquad \text{(at constant temperature)}$$

Which is Boyle's law.

DEDUCTION OF CHARLE'S LAW:

From kinetic molecular theory

$$P = \frac{1}{3} \ell \overline{V^2}$$

or $$P = \frac{1}{3} mn_v \overline{V^2} \qquad [\ell = mn_v]$$

or $$P = \frac{1}{3} m \frac{N}{V} \overline{V^2} \qquad [n_v = N/V]$$

or $$PV = \frac{1}{3} mN\overline{V^2}$$

or $$PV = \frac{2}{3} N \frac{1}{2} m\overline{V^2} \qquad [\tfrac{1}{2} mV^2 = (K.E.)_{Avg} = 3/2 . KT]$$

or $$PV = \frac{2}{3} N \frac{3}{2} KT$$

or $$PV = N (KT)$$

or $$\frac{V}{T} = \frac{KN}{P}$$

Now if 'P' is kept constant then (KN/P) becomes constant

$$\boxed{\frac{V}{P} = \text{constant}}$$ (at constant pressure)

Which is Charle's law.

HEAT CAPACITY:

When temperature of a body is increased from T_1 to T_2 then increase in temperature is proportional to the heat absorbed by the body (ΔQ).

i.e. $\Delta Q \ \alpha \ \Delta T$

or $\Delta Q = (\text{constant})\ \Delta T$

or $\Delta Q = c' \ \Delta T$

or $$\boxed{c' = \frac{\Delta Q}{\Delta T}}$$

Here 'c′' is the constant 'Heat Capacity'. It is defined as "The amount of heat required to raise the temperature of a body by one degree centigrade or Kelvin."

Unit: In M.K.S system its units $J.K^{-1}$.

SPECIFIC HEAT CAPACITY:

If a body of mass 'm' is provided amount of heat 'ΔQ' to rise its temperature 'ΔT' then the amount of heat absorbed is proportional to the mass of the body and rise in temperature.

i.e. $\Delta Q \ \alpha \ \Delta T$

& $\Delta Q \ \alpha \ m$

Combining these factors

$\Delta Q \ \alpha \ m \ \Delta T$

or $\Delta Q = (\text{constant})\ m \ \Delta T$

or $\Delta Q = c \ m \ \Delta T$

or $$\boxed{c = \frac{\Delta Q}{m\Delta T}}$$

Here 'c' is constant of proportionality and known as 'Specific Heat Capacity' and it is defined as "amount of heat required to rise the temperature of unit mass of a body by one degree centigrade or Kelvin."

Unit: In M.K.S system its unit is $J.kg^{-1}.K^{-1}$

Further $c = \frac{\Delta Q}{\Delta T} \times \frac{1}{m}$ [$\Delta Q/\Delta T = c'$]

$$\boxed{c = \frac{c'}{m}}$$

MOLAR HEAT CAPACITY:

We know that

$$\text{Specific heat capacity}(c) = \frac{\Delta Q}{m\Delta T} \rightarrow \text{(i)}$$

we know that

$$\text{no. of moles } (n) = \frac{\text{Mass in gm (m)}}{\text{Molecular Mass (M)}}$$

or $$m = nM$$

Putting in Equation (i)

$$c = \frac{\Delta Q}{nM\Delta T}$$

or $$Mc = \frac{\Delta Q}{n\Delta T}$$

or $$\boxed{C = \frac{\Delta Q}{n\Delta T}}$$

Here the product Mc is denoted by C (Molar Specific Heat Capacity) and it defined as the amount of heat required to rise the temperature of one mole of a substance by one degree Kelvin or centigrade.

Unit: In M.K.S system its unit is $J.mole^{-1}.K^{-1}$.

LATENT HEAT:

"Latent heat is the amount of heat required to change the state of a substance without changing its temperature."

SPECIFIC LATENT HEAT OF FUSION (H_f):

"The amount of heat required to convert unit mass of a solid substance completely into liquid at its melting point without change of temperature is called its latent heat of fusion (H_f)."

$$\boxed{H_f = \frac{\Delta Q}{m}}$$

Example: Latent heat of fusion for ice $H_f = 3.36 \times 10^5$ J/kg.

Unit: In M.K.S system its unit is J/kg or $J.kg^{-1}$.

SPECIFIC LATENT HEAT OF VAPOURIZATION (H_v):

"The amount of heat required to convert unit mass of a liquid substance completely into vapour at its boiling point without changing its temperature."

$$\boxed{H_v = \frac{\Delta Q}{m}}$$

Example: Latent heat of vapourization for water $H_v = 2.26 \times 10^6$ J/kg.

Unit: In M.K.S system its unit is J/kg or $J.kg^{-1}$.

THERMODYNAMICS:

"Thermodynamics is the branch of physics which deals with the transfer of heat energy into mechanical energy (work)."

SOME IMPORTANT THERMODYNAMIC TERMS

SYSTEM:

A collection of matter having well defined boundaries is called system.

OPEN SYSTEM:

"A system through which the transfer of mass and energy both is possible called an open system."

Example: A human body.

CLOSED SYSTEM:

"A system through which the transfer of energy is possible but transfer of mass is not possible is called a closed system."

Example: Hot water enclosed in a test tube tightly.

ISOLATED SYSTEM:

"A system through which the transfer of both mass & energy is not possible, is called an isolated system."

Example: A thermos.

HEAT RESERVOIR:

"It is a body of large mass i.e. huge heat capacity and its temperature does not change by absorption or removal of a little amount of heat."

HEAT ENGINE:

A heat engine is such a device which converts the heat energy into mechanical energy.

A heat engine consists of following three parts.

1) A Source (A heat reservoir at higher temperature T_1)
2) A Sink (A cold reservoir at a low temperature T_2)
3) Working Substance.

FIRST LAW OF THERMODYNAMICS:

According to first law of thermodynamics the difference of heat energy provided to a system (ΔQ) and the work done (ΔW) is always equal to the change in internal energy (ΔU) of the system independent of the path followed.

i.e. $\Delta Q - \Delta W = \Delta U$

or $\boxed{\Delta Q = \Delta U + \Delta W}$

In other words according to first law of thermodynamics it is impossible to construct such a device which produces work done more than heat energy provided to it which is another form of law of conservation of energy which states that the total energy of an isolated system always remains constant.

INTERNAL ENERGY:

It is a Microscopic property of a system and it may be defined as the sum of all energies of all the molecules of a system.

It is impossible to find out absolute internal energy of a system, however the change in internal energy of a system can be determined by calculating the difference of heat provided & work done by a system.

i.e. $\Delta U = \Delta Q - \Delta W$

or $\boxed{U_2 - U_1 = \Delta Q - \Delta W}$

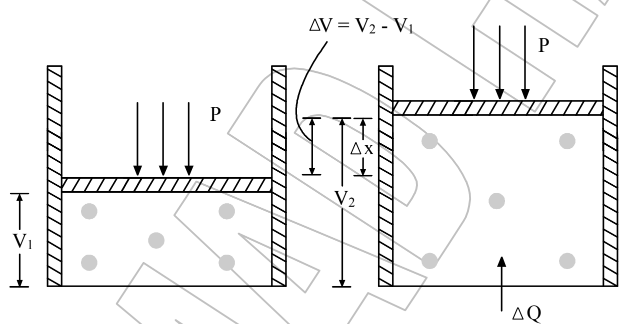
APPLICATIONS OF FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics can be studied in different thermodynamic processes.

1) ISOBARIC PROCESS:

"A process of thermodynamics in which the pressure remains constant is called an isobaric Process."

Lets us consider a cylinder having and ideal gas enclosed by a moveable and frictionless piston of cross-sectional area (A).



When an amount of heat (ΔQ) is provided to the gas in the cylinder then it expands and the piston moves a distance (Δx) as to keep the pressure constant and a change in volume (ΔV) of gas occurs. Therefore from the figure

Work done = F . d

or $W = F \cdot \Delta x \quad \rightarrow$ (i)

We know that

$P = \dfrac{F}{A}$ (P = Pressure)

& $F = PA$

Putting in equation (i)

$\Delta W = PA(\Delta x)$

or $\Delta W = P(A\Delta x)$

but $\Delta V = A\Delta x$

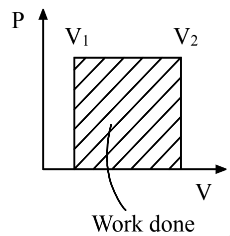
$\Delta W = P\Delta V$

According to first law of thermodynamics

$$\Delta Q_p = \Delta W + \Delta U$$

or $$\boxed{\Delta Q_p = P\Delta V + \Delta U}$$

It shows that the heat absorbed at constant pressure is a characteristics property of the system i.e. $\Delta Q_p = P\Delta V + \Delta U$ which known as enthalpy (ΔH).



Beside it also indicates that the internal energy of the system may be changed by either doing work or by providing heat to the system.

2) <u>ISOCHORIC PROCESS:</u>

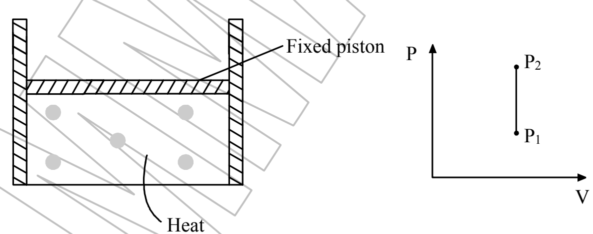
"A process that takes place at constant volume is called an isochoric process."

Let us consider a cylinder having some amount of a gas and a fixed piston. When an amount of heat (ΔQ) is provided to the cylinder which contains insulated walls and piston, then no work is done since volume is constant and as

$$\Delta W = P\Delta V$$

$$\Delta W = P\,(0) \qquad [\,\Delta V = 0\,]$$

$$\Delta W = 0$$



Applying first law of thermodynamic

$$\Delta Q = \Delta U + \Delta W$$

But $\Delta W = 0$

$$\Delta Q = \Delta U + 0$$

or $$\boxed{\Delta Q = \Delta U}$$

This result shows that the heat provided to a system at constant volume does not do any work but it is all utilized in increasing the internal energy of the system.

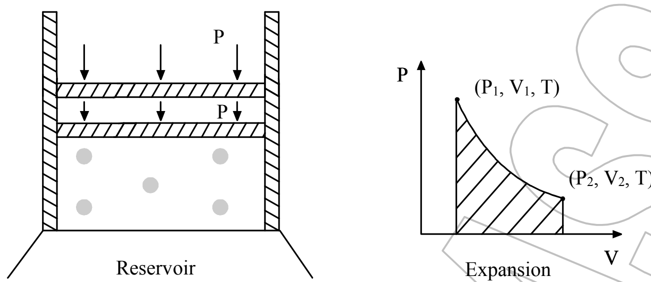
3) <u>ISOTHERMAL PROCESS:</u>

"A process which takes place at constant temperature is called an isothermal process."

Let us consider a cylinder with insulated walls and insulated, moveable and frictionless piston and having some gas enclosed. To keep its temperature constant the base of cylinder is kept on a heat reservoir and the process is performed very slowly.

<u>Isothermal Expansion:</u>

Let the gas in the cylinder expand by decreasing load on the piston. As the piston moves upward the temperature of the gas falls and to overcome it, some heat flows from the reservoir & hence the temperature remains constant. In this process $\Delta U = 0$ because temperature is constant.



Isothermal Expansion

Applying first law of thermodynamics

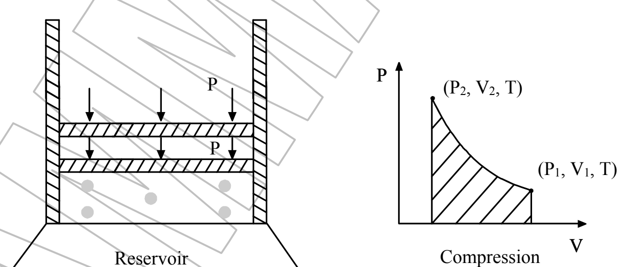
$$\Delta Q = \Delta W + 0$$

or $$\boxed{\Delta Q = \Delta W}$$

It shows that the amount of heat provided isothermally, all is utilized in work done by the system.

<u>Isothermal Compression:</u>

If the gas is allowed to be compressed by increasing load on the piston then the temperature rises and therefore hear transfer into reservoir and the temperature remains constant.



Isothermal Compression

In this case

$$\boxed{-\Delta Q = -\Delta W}$$

i.e. Heat rejected by the system is equal to the work done on the system.

4) <u>ADIABATIC PROCESS:</u>

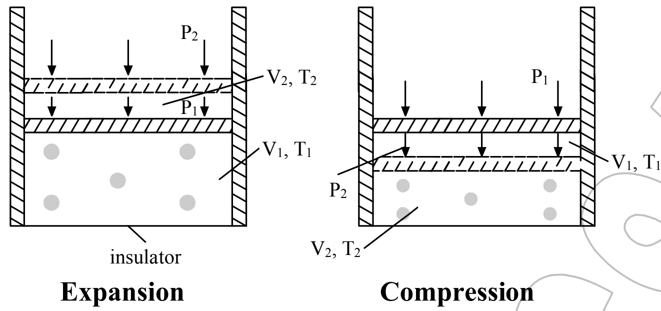
"A process during which heat does not flow into or out of the system is called an adiabatic process."

Let us consider a cylinder having some gas with insulated walls and an insulated moveable frictionless piston. Its base is either insulated or it is kept on a heat insulator.

Adiabatic Expansion:

When the pressure or load on the piston is decreased then it moves upwards and the gas expands. The temperature falls down and a work is done by the system. Here heat transfer is zero therefore

$\Delta Q = 0$



Applying first law of thermodynamics

$$\Delta Q = \Delta W + \Delta U$$

or $$0 = \Delta W + \Delta U$$

or $$\boxed{\Delta W = -\Delta U}$$

The above equation shows that the work by the system is done on the cost of loss of internal energy of the system.

Adiabatic Compression:

When the load on the piston increases then the work is done on the system and the temperature of the gas increases.

Here applying first law of thermodynamics

$$\Delta Q = \Delta W + \Delta U$$

or $$0 = \Delta W + \Delta U$$

or $$\boxed{\Delta U = -\Delta W}$$

It is clear from the above equation that the work done on the system increases the internal energy of the system.

MOLAR HEAT CAPACITY:

There are two important types of molar heat capacity.

1) Molar heat capacity at constant pressure (C_p).
2) Molar heat capacity at constant volume (C_v).

1) MOLAR HEAT CAPACITY AT CONSTANT PRESSURE (C_p):

"The amount of heat required to increase the temperature of one mole of a substance by 1K, at constant pressure is called molar specific heat at constant pressure and it is denoted by 'C_p'."

$$\boxed{C_p = \frac{\Delta Q_p}{n\Delta T}}$$

2) MOLAR HEAT CAPACITY AT CONSTANT VOLUME (C_v):

"It is the amount of heat required to increase the temperature of one mole of a substance by 1K at constant volume. It is denoted by C_v."

$$\boxed{C_v = \frac{\Delta Q_v}{n\Delta T}}$$

EXPLAIN WHY $C_p > C_v$:

Suppose a gas in a cylinder is provided heat at (i) constant volume & (ii) at constant pressure. The heat supplied at constant volume is all utilized to increase K.E. of molecules or temperature of the gas, whereas the heat supplied at constant pressure is partially used to increase the temperature of gas and for work done. Therefore for same rise of temperature the amount of heat provided at constant pressure should be more than the heat supplied at constant volume.

i.e. $\Delta Q_p > \Delta Q_v$

or $n\Delta T C_p > n\Delta T C_v$ $[C_p = \Delta Q_p / n\Delta T \quad \& \quad C_v = \Delta Q_v / n\Delta T]$

or $\boxed{C_p > C_v}$

PROVE THAT $C_p - C_v = R$

Consider a cylinder with a frictionless and moveable piston containing 'n' moles of an ideal gas.

Now first of all consider a process at constant volume. When gas temperature is raised to a temperature difference of ΔT then heat absorbed by the system.

$\Delta Q_v = n\ \Delta T\ C_v \rightarrow$ (i)

Applying first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

But $\Delta W = P\Delta V$ and $\Delta V = 0$ [Volume is constant]

$\Delta W = 0$

or $\Delta Q_v = \Delta U \rightarrow$(ii)

Now heat the gas at constant pressure then the amount of heat absorbed for the same rise of temperature (ΔT).

$\Delta Q_p = n\ \Delta T\ C_p \rightarrow$ (iii)

Now applying first law of thermodynamics

$\Delta Q_p = \Delta U + \Delta W$

or $\Delta Q_p = \Delta U + P\Delta V \rightarrow$ (iv) $[\Delta W = P\Delta V]$

But from equation (ii) $\Delta U = \Delta Q_v$

Equation (iv) becomes

$\Delta Q_p = \Delta Q_v + P\Delta V \rightarrow$ (v)

Now putting the values of ΔQ_v & ΔQ_p from equation (i) & (iii)\

$$n\Delta TC_p = n\Delta TC_v + P\Delta V$$

or $$n\Delta TC_p - n\Delta TC_v = P\Delta V$$

or $$n\Delta T\,(C_p - C_v) = P\Delta V \rightarrow \text{(vi)}$$

Applying general gas equation

$$PV_1 = nRT_1 \quad \rightarrow \text{(vii)} \qquad \text{[P is constant]}$$

& $$PV_2 = nRT_2 \quad \rightarrow \text{(viii)} \qquad \text{[P is constant]}$$

Subtracting equation (vii) from (viii)

$$\begin{array}{l} PV_2 = nRT_2 \\ \underline{PV_1 = nRT_1} \\ -\quad\;\; - \\ PV_2 - PV_1 = nRT_2 - nRT_1 \end{array}$$

or $$P\,(V_2 - V_1) = nR\,(T_2 - T_1)$$

or $$P\Delta V = nR\Delta T \rightarrow \text{(ix)}$$

Putting in equation (vi)

$$n\Delta T\,(C_p - C_v) = nR\Delta T$$

or $$\boxed{C_p - C_v = R} \qquad \text{Proved}$$

SECOND LAW OF THERMODYNAMICS

This law was presented by Lord Kelvin and Rudolf Clausius in 1850. This law can be stated according to the following two statements.

1) CLAUSIUS STATEMENT:

Clausius says that "It is impossible to make heat flow from a cold body to a hot body without the expenditure of energy or work."

2) KELVIN'S STATEMENT:

According to this statement "It is impossible to construct such an engine working in cycles continuously (Perpetual Machine of second kind) which derives heat from a source and perform an equivalent amount of work without rejecting any heat to a body at lower temperature (sink).

According to this statement for the working of a heat engine, two bodies at different temperatures are necessary.

EQUIVALANCE OF CLAUSIUS AND KELVIN'S STATEMENT:

Suppose Kelvin's statement is false and we can construct a heat engine which converts all the heat absorbed from a source into work completely. If this perfect engine is connected to a refrigerator that conveys heat from a cold body to a hot body. The net result is a transfer of heat from cold to hot body without expenditure of energy which is the violation of clausius statement.

Therefore clausius statement becomes wrong if Kelvin's statement is taken as wrong. Therefore we can say that both the statements facilitate each other.

THE CARNOT ENGINE:

The Carnot Engine was designed by Sadi Carnot in order to set an ultimate limit of the efficiency of a heat engine in 1824. It was an ideal engine which was free from all sorts of heat losses and friction. It consists of

(a) A gas cylinder with perfectly insulating walls and perfectly conducting base.
(b) A perfectly insulated, weightless and frictionless piston in the cylinder.
(c) One mole of an ideal gas as working substance.

CARNOT CYCLE:

A cycle of a heat engine completes when the properties of a system returned to the original state. The operating cycle of a carnot engine is called Carnot cycle which consists of the following four processes:

1) Isothermal Expansion 2) Adiabatic Expansion
3) Isothermal Compression 4) Adiabatic Compression

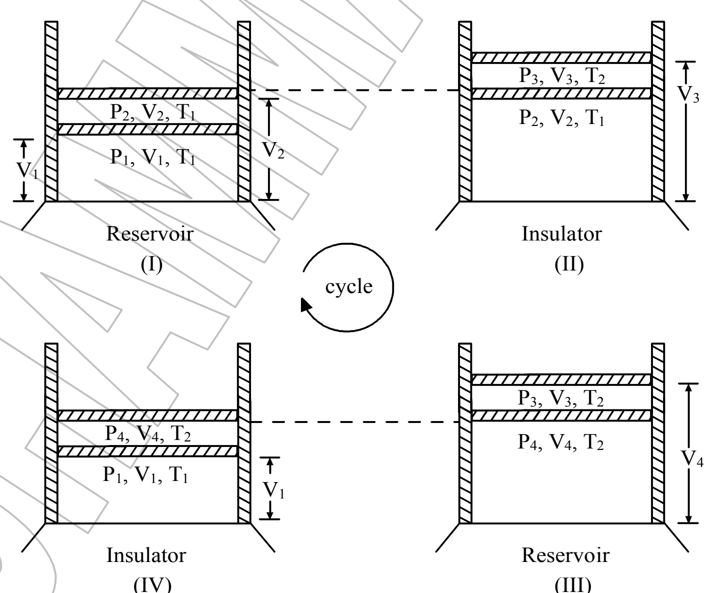
1) ISOTHERMAL EXPANSION:

Consider a carnot engine. Let the working substance be at pressure P_1, volume V_1 and temperature T_1. Let the engine be placed on a heat reservoir of temperature T_1.

If the gas is allowed to expand (by decreasing load on the piston), some amount of heat Q_1 is absorbed by the gas through the base of the cylinder, keeping the temperature constant. However the gas does work on the piston. The volume increases and pressure decreases therefore the initial state P_1, V_1, T_1 becomes P_2, V_2, T_1 with absorption of heat (Q_1).

2) ADIABATIC EXPANSION:

If the engine is placed on an insulator, then no heat can enter or leave the system. If the gas is allowed to expand till the volume increases from V_2 to V_3. The kinetic energy of the molecules decreases and work is done at the cost of internal energy. Thus temperature falls from T_1 to T_2, and pressure decreases from P_2 to P_3. The change in state is form P_2, V_2, T_1 to P_3, V_3 T_2.



3) ISOTHERMAL COMPRESSION:

Let the engine be placed on a heat reservoir (at temperature T_2). Now the gas is compressed by increasing the load on piston then the temperature of system increases and to keep it constant an amount of heat Q_2 is rejected into the reservoir. The piston does work on the gas. The volume decreases from V_3

to V_4 and pressure increases from P_3 to P_4 at constant temperature T_2. The change of state is from P_3, V_3, T_2 to P_4, V_4, T_2. An amount of heat Q_2 is also rejected.

4) ADIABATIC COMPRESSION:

Let the engine be placed on an insulating medium. The gas is compressed adiabatically to its original state. Work is done and internal energy increases and thus temperature increases. The system turns to its original state P_1, V_1, T_1.

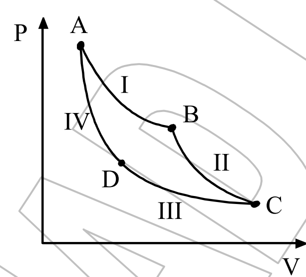
During the whole cycle, carnot engine performs a net amount of work, ΔW, which is the difference between the work done on the engine during two expansions and two compressions. The net amount of heat absorbed by the engine is $Q_1 - Q_2$ in one cycle, where Q_1 is heat absorbed during the isothermal expansion and Q_2 is the heat rejected during the isothermal compression.

EFFICIENCY OF CARNOT ENGINE:

Thermal efficiency of a heat engine is the ratio of output to input.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work obtained}}{\text{Heat supplied}}$$

i.e $$\boxed{\eta = \frac{\Delta W}{Q_1}}$$



Carnot Cycle

or $$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}$$

or $$\eta = 1 - \frac{Q_2}{Q_1} \quad \rightarrow \text{(i)}$$

Practically it can be proved that ratio of heat rejected and heat supplied is equal to the ratio of temperature of sink and temperature of source.

i.e. $$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\boxed{\eta = 1 - \frac{T_1}{T_2}}$$

or $$\boxed{\%\,\eta = \left(1 - \frac{T_1}{T_2}\right) \times 100}$$

The above equations shows that the efficiency of carnot engine is 100% if and only if the temperature of sink (T_2) is the absolute zero (0K). But, according to the third law of thermodynamics, the absolute zero carnot be achieved by any finite series of operations.

The efficiency of carnot engine which is an upper limit of the performance of engine is less than 100% therefore no practical engine can be made 100% efficient any way.

ENTROPY:

The change of entropy (ΔS) is defined as the quantity of heat (ΔQ) added to or removed from a system divided by the Kelvin temperature (T).

Thus $\boxed{\Delta S = \frac{\Delta Q}{T}}$

It units is JK^{-1}.

Entropy does not change during an adiabatic process.

ENTROPY AS DISORDER OF SYSTEM:

"We observe that the numerical value of entropy increases in all natural process, provided the system is considered as a whole. It is also observed that in every natural process, the disorder of a system insurances. Thus we can concluded that entropy is a measure of disorder of a system."

ENTROPY AS UNAVAILABILITY OF ENERGY:

If one tank of hot water and one tank of cold water are mixed, the ordered arrangement is lost, and the energy of the system attains a state when it can not be converted into useful work. Whole of the water is at some temperature. No energy is available from such a system. Thus, increase of entropy is a measure of unavailability of energy of a system.

LAW OF INCREASE OF ENTROPY:

It seems to be a law of nature that all natural process always take place in such a direction so as to cause an increase in the entropy of a system and its surroundings. Thus the second law of thermodynamics is also expressed as law of increase of entropy. It states that

"The entropy of the universe during any process either remains constant or increases."

$\boxed{\text{Symbolically} \quad \Delta S \geq 0}$

XII PHYSICS

NOTES

CHAPTER # **12**

ELECTROSTATICS

ELECTRIC CHARGE:

Electric charge can be defined as the property due to which two objects attract or repel each other.

There are two types of electric charge i.e. positive & negative. “unlike charges attract & like charges repel each other”.

Charge is always quantized i.e. $\boxed{q = ne}$

Where,

q = electric charge & e = charge of an electron & n = any integer (1, 2, 3, 4, ………)

COULOMB’S LAW:

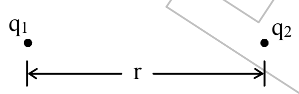
In 1785, a French Scientist named Charles Augustine development coulomb developed a law regarding force of attraction or repulsion between two point charges.

Statement:

“The force of attraction or repulsion between two point charges is directly proportional to the product of their charges & inversely proportional to square of distance between them.”

Explanation:

Consider two point charges q_1 & q_2 placed at a distance ‘r’.



Mathematically, electric force

$$F \propto q_1 q_2$$

& $$F \propto 1/r^2$$

combining these factors

$$F \propto \frac{q_1 q_2}{r^2}$$

or $$\boxed{F = K \frac{q_1 q_2}{r^2}}$$

Where K is the constant of proportionality & its value depends on the nature of medium between charges & system of units.

In M.K.S system for free space (vacuum) as medium the value of K is

$$K = \frac{1}{4\pi\varepsilon_o}$$

Where ‘ε_o’ is the permittivity of free & its value is $8.85 \times 10^{12}\ C^2.N^{-1}.m^{-2}$.
The value of $K = 8.98 \times 10^9$ or $9 \times 10^9\ N.m^{-2}.C^{-2}$.

Coulomb law may also be written as

$$\boxed{F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2}}$$

In vector form

$$\boxed{F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2} \hat{r}}$$

Where 'r^' is the unit vector which specifies the direction of force.

If dielectric is used as medium i.e. the medium between the charges is other than air (free space) then the electrostatic force decreases by a 'ε_r' called relative permittivity of the medium and it is also known as dielectric constant.

Thus

$$F' = \frac{F}{\varepsilon_r}$$

or $$F' = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \frac{q_1 q_2}{r^2}$$

Further $\varepsilon_r = \varepsilon / \varepsilon_o$ where ε is permittivity of the medium

$$\varepsilon_o \varepsilon_r = \varepsilon$$

or $$\boxed{F' = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}}$$

Coulomb (C):

Coulomb is the S.I unit of electric charge. 1 coulomb charge is equal in value to the charge contained in 6.25×10^{18} electrons i.e. $1C = 6.25 \times 10^{18}$ e

Where 'e' = charge of one electron

"1 Coulomb is defined as the charge appeared on two point charges when they are 1m apart in free space & exert a force of 9×10^9 N on each other."

ELECTRIC FIELD:

"The space or region around an electric charge within which its effect can be felt is called electric field. Electric field is the quantity."

ELECTRIC FIELD INTENSITY (E):

Electric field intensity of a charge at a particular point in its field is defined as the force experienced by a unit positive test charge at that point.

i.e. $$E = \frac{F}{q_o}$$

The S.I unit of E is NC^{-1}. (Newton per Coulomb)

According to coulomb law the electrostatic force between two charges is

$$F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r^2}$$

Therefore the force of a charge 'q' at a distance 'r' on a +vector test charge 'q_o' will be

$$F = \frac{1}{4\pi\varepsilon_o} \frac{q\, q_o}{r^2}$$

& Electric field intensity at that point will be

$$E = \frac{F}{q_o} = \frac{1}{4\pi\varepsilon_o}\,\frac{q\,q_o}{r^2} \times \frac{1}{q_o}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_o}\,\frac{q}{r^2} = K\frac{q}{r^2}}$$

For dielectric as medium

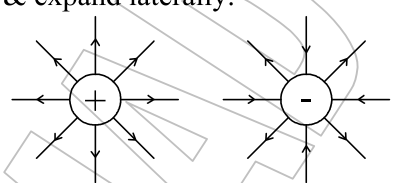
$$\boxed{E = \frac{1}{4\pi\varepsilon_o}\,\frac{q}{r^2}}$$

ELECTRIC FIELD LINES:

Electric field lines are imaginary lines drawn in electric field such direction of the tangent at any point represents the direction of electric field at that point. The density (number of lines per unit area) represents the strength of magnetic field.

Properties:

1) The electric field lines are imaginary lines but the filed they represent is read.
2) The electric field lines originate from a positive charge and terminate at a negative charge.
3) They can not pass through a conductor.
4) They never interest each other.
5) These lines are continuous.
6) The field due to a point charge has spherical symmetry.
7) They contract longitudinally & expand laterally.



ELECTRIC FLUX:

Electric flux is a scalar property of vector field. It is defined as the dot product of electric intensity and vector area. i.e.

$$\boxed{\Delta\text{Ø} = E\,.\,\Delta A}$$

The unit of electric flux is $N.m^2.C^{-1}$.

The electric flux depends on intensity, area and orientation of the surface relative to the field.

FLUX DENSITY:

"The flux density is defined as the electric flux passing through unit area."

$$E = \frac{\Delta\text{Ø}}{\Delta A}$$

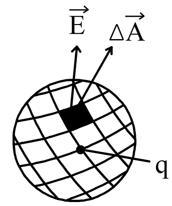
Flux density, in fact, represents the electric field intensity at any point. Flux density, therefore, gives the no. of field lines originating from a charge through unit area.

$$\boxed{\text{Flux density} = \frac{\text{No. of lines originating from charge}}{\text{Surface Area}}}$$

FLUX THROUGH A SPHERE DUE TO CHARGE AT CENTRE:

Consider a hollow sphere of radius 'r' having a charge +q placed at the centre.

Now let us divide the surface area of the sphere into 'n' small and equal patches, each of area ΔA. The lines of force spread uniformly and radially. The flux over the surface of sphere is



$$\O_e = \Sigma\Delta\O_e$$

Where '$\Delta\O_e$' is flux passing through each patch.

or $\O_e = \Sigma(E.\Delta A)$ $[\O_e = E.\Delta A]$

As the angle between E & A is zero

$$\O = \Sigma E\Delta A \cos 0^o$$

or $\O = \Sigma E\Delta A$

or $\O = E\Sigma\Delta A$

& $\O = E$ (Area of Spherical Surface)

$$\O_e = E \times 4\pi r^2$$

But $E = \dfrac{1}{4\pi\varepsilon_o} \dfrac{q}{r^2}$

$$\O_e = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r^2} \times 4\pi r^2$$

or $\boxed{\O_e = \dfrac{q}{\varepsilon_o}}$

Therefore the above expression shows that the flux through sphere does not depend upon size of the sphere.

GAUSS' LAW:

Gauss' law gives a relationship between electric flux and net charge enclosed by a surface.

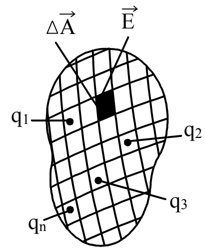
Statement:

"The total electric flux through any closed surface is $1/\varepsilon_o$ times the total charge enclosed in it."

Proof:

Consider a closed surface of some arbitrary shape. Let it encloses point charges q_1, q_2, q_3, ……., q_n.

Total charge enclosed = $q_1 + q_2 + q_3 + \ldots\ldots + q_n$ → (i)



Now to calculate the electric flux through the surface, it is divided into very small but equal parts. Each part is almost plane therefore the direction of E & ΔA is same.

It is obvious that the total flux passing through the surface must be equal to the algebraic sum of the flux through all patches.

i.e. $\quad Ø = Ø_1 + Ø_2 + Ø_3 + \ldots\ldots\ldots + Ø_n$

Consider flux '$Ø_1$' due to charge q_1. If we imagine a sphere having q at its centre then flux through this imaginary sphere and the surface will be the same.

i.e. $\quad$ flux due to $q_1 = Ø_1 = \dfrac{q_1}{\varepsilon_o}$

Similarly,

flux due to $q_2 = Ø_2 = \dfrac{q_2}{\varepsilon_o}$

$\vdots \qquad \vdots \qquad \vdots$

& $\quad$ flux due to $q_n = Ø_n = \dfrac{q_n}{\varepsilon_o}$

Total flux $(Ø) = Ø_1 + Ø_2 + Ø_3 + \ldots\ldots\ldots + Ø_n$

$$Ø = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} + \frac{q_3}{\varepsilon_o} + \ldots\ldots\ldots + \frac{q_n}{\varepsilon_o}$$

or
$$Ø = \frac{1}{\varepsilon_o}(q_1 + q_2 + q_3 + \ldots\ldots + q_n)$$

&
$$\boxed{Ø = \frac{1}{\varepsilon_o}(\text{Total charge enclosed})}$$

Thus it is proved that the electric flux through the surface is $1/\varepsilon_o$ times of the total charge enclosed.

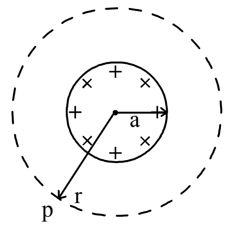
APPLICATIONS OF GAUSS' LAW:

Gauss' law can be applied to evaluate electric field intensity by the proper choice of Gaussian surface.

CASE: I(a)

Electric field intensity of a charged sphere at a point outside the sphere:

Consider a sphere with radius 'a' and a charge 'q' uniformly distributed on its surface. If 'P' is a point outside the sphere at a distance 'r' from the centre 'O' then imagine a Gaussian surface as shown in the figure.



Because of spherically symmetric charge distribution, the field has the same magnitude with respect to every point on the closed surface.

$$\text{Ø} = \Sigma E.\Delta A$$

& $$\text{Ø} = \Sigma E \Delta A \cos 0^o$$

$$\text{Ø} = \Sigma E \Delta A \qquad [\cos 0^o]$$

or $$\text{Ø} = E \Sigma A$$

or $$\text{Ø} = E(4\pi r^2) \quad \rightarrow \text{(i)} \qquad [\text{surface area} = 4\pi r^2]$$

According to Gauss' law

$$\text{Ø} = \frac{q}{\varepsilon_o} \quad \rightarrow \text{(ii)}$$

Comparing (i) & (ii)

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_o}$$

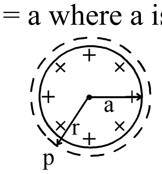
or $$\boxed{E = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r^2}\right)}$$

This indicates that the field outside the charged sphere behaves as the all charge is concentrated at its centre.

CASE: I(b)

Electric field intensity of a charged sphere on its surface:

To evaluate intensity at a point on the surface of charged sphere the field is taken at a point outside but infinitely close to the sphere i.e. r = a where a is the radius of sphere;



Field at a distance 'r' from a charged sphere is

$$E = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r^2}\right)$$

But $r \rightarrow a$

$$E = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{a^2}\right)$$

If charge density (σ) is the charge per unit area.

i.e. $\sigma = \dfrac{q}{A}$

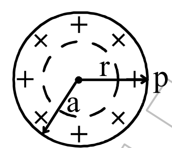
Then $\sigma = \dfrac{q}{4\pi a^2}$ $[A = 4\pi r^2 = 4\pi a^2 \quad :a = r]$

or $\boxed{E = \dfrac{\sigma}{\varepsilon_o}}$

CASE: I(c)

Electric field intensity at a point inside the charged surface:

Now if we consider a point inside the charged surface then the Gaussian surface has a radius less then the radius of charged sphere and therefore it encloses no charge.



Since q = 0
Hence flux should be zero

& $E = \dfrac{1}{4\pi\varepsilon_o} \dfrac{(0)}{a^2}$

or $\boxed{E = 0}$

CASE: II

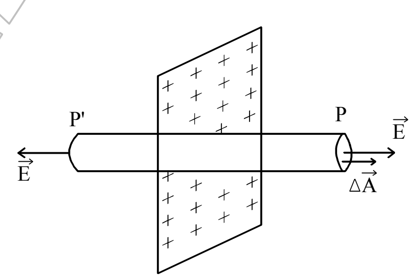
Electric field intensity due to an infinite sheet of charge:

Consider a non-conducting sheet of infinite extent & which is charged uniformly.

Now if charge density, $\sigma = \dfrac{q}{\Delta A}$

Then total charge q = σ ΔA

Now consider a Gaussian surface in the form of a cylinder of cross section ΔA. The surface emerges out perpendicularly on both sides. It is observed that flux does not pass from curved surface of the cylinder. But, in fact, it passes through both the ends (plane surfaces) of cross-sectional area ΔA of the cylinder.



The total flux

$\varnothing = 2\,(E.\Delta A)$

or $\quad \text{Ø} = 2E\Delta A\cos\theta \quad$ [since sum of flux on both sides of plate E.ΔA + E.ΔA]

But E is parallel to the vector area

$$\text{Ø} = 2E\Delta A\cos 0^o \qquad [\cos 0^o = 1]$$

$$\text{Ø} = 2E\Delta A \quad \rightarrow \text{(i)}$$

According to Gauss' law

$$\text{Ø} = \frac{q}{\varepsilon_o} \quad \rightarrow \text{(ii)}$$

Comparing (i) & (ii)

$$2E\Delta A = \frac{q}{\varepsilon_o}$$

But $q = \sigma\Delta A$

$$2\Delta A \times E = \frac{\sigma\Delta A}{\varepsilon_o}$$

or

$$\boxed{E = \frac{\sigma}{2\varepsilon_o}}$$

CASE: III

Electric field intensity between two oppositely charged plates:

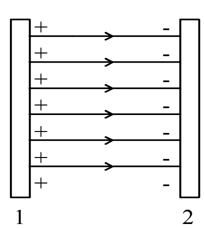
Consider two plates containing equal amounts of opposite charges.
Let the surface charge density of plate 1 is + σ
Let the surface charge density of plate 2 is - σ

Electric field intensity at point 'P' due to plate 1 will be

$$E_1 = \frac{\sigma}{2\varepsilon_o}$$

& Electric field intensity at point 'P' due to plate 2 will be

$$E_2 = \frac{\sigma}{2\varepsilon_o}$$



Total electric field intensity

$$E = E_1 + E_2$$

or $$E = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o}$$

$$E = \frac{\sigma + \sigma}{2\varepsilon_o}$$

$$E = \frac{2\sigma}{2\varepsilon_o}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_o}}$$

ELECTRIC POTENTIAL:

"In an electric field the potential difference between two points is defined as the work done in moving a unit positive charge from one point to another against the electric field."

If charge q_o is moved from P to Q then P.d. of Q with respect to P is

$$V_Q - V_P = \frac{W_{PQ}}{q_o}$$

or $$\boxed{\Delta V = \frac{\Delta W}{q_o}}$$

S.I unit of electric potential is Volt (V) & $J.C^{-1} = 1\ V$

If the work done against field between two points to move a unit charge from point to another is 1 J, then electric potential is called 1 volt (1V).

RELATION BETWEEN ELECTRIC POTENTIAL & INTENSITY:

If a charge q_o is moved from P to Q against the electric field then

$$V_Q - V_P = \frac{W_{P \to Q}}{q_o}$$

or $$\Delta V = \frac{F.\Delta r}{q_o} = \frac{q_o E.\Delta r}{q_o}$$ [E = f/q :F =qE]

$$\Delta V = E.\Delta r$$

or $$\Delta V = E\Delta r\cos 180^o$$

or $$\Delta V = -E\Delta r$$

or $$\boxed{E = \frac{-\Delta V}{\Delta r}}$$

From the above equation the unit of intensity becomes

$$E = V.m^{-1} \text{ (volt/meter)}$$

Furthermore;

$$\frac{V}{m} = \frac{J}{Cm} = \frac{N.m}{C.m} = NC^{-1}$$

$$\boxed{V.m^{-1} = N.C^{-1}}$$

ELECTRIC POT DIFF DUE TO AN ISOLATED POINT CHARGE BET TWO FAR OFF POINTS:

Consider an isolated point charge ‘q’. Now consider two points A & B in a straight line which are far away from each other. Let a test charge ‘q_o’ placed at A.

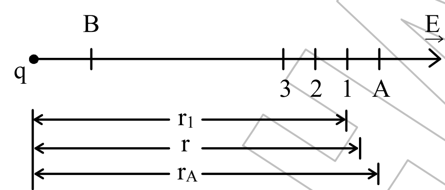
In this situation the potential difference between points ‘A’ & ‘B’ can not be evaluated using constant throughout (as $E \propto 1/r^2$). Therefore for this purpose we divide the distance into n steps between A & B. the distance Δr between two points is kept so small that the electric field intensity between two points remains nearly constant.

The net potential difference between points A & B can be calculated as following;

$$V_{A\to B} = V_{A\to 1} + V_{1\to 2} + V_{2\to 3} + \ldots\ldots\ldots\ldots + V_{n\to B} \quad \to \text{(i)}$$

Now to calculate $V_{A\to 1}$

Let distance from the source charge to position A = r_A
Let distance from the source charge to position 1 = r_1



Length of interval = $\Delta r = r_1 - r_A$
Distance between the point (Geometric Mean) = $r = \sqrt{r_A r_1}$ $\quad \to$ (ii)

To prove that $r = \sqrt{r_A r_1}$

Since $\quad r = \dfrac{r_A + r_1}{2}$

Squaring both sides

$$(r)^2 = \left(\frac{r_A + r_1}{2}\right)^2$$

$$r^2 = \left(\frac{r_A + r_A + \Delta r}{2}\right)^2$$

$$r^2 = \left(\frac{2r_A + \Delta r}{2}\right)^2$$

$$r^2 = \left(\frac{2r_A}{2} + \frac{\Delta r}{2}\right)^2$$

$$r^2 = \left(r_A + \frac{\Delta r}{2}\right)^2$$

$$r^2 = r_A^2 + 2r_A.\frac{\Delta r}{2} + \left(\frac{\Delta r}{2}\right)^2$$

Neglecting $\left(\dfrac{\Delta r}{2}\right)^2$

$$r^2 = r_A^2 + r_A\Delta r$$

or $r^2 = r_A^2 + r_A(r_1 - r_2)$ $[\Delta r = r_1 - r_2]$

or $r^2 = r_A^2 + r_A r_1 - r_n^2$

$\sqrt{r^2} = \sqrt{r_A r_1}$

$r = \sqrt{r_A r_1}$

Now p.d. ΔV or $V_{A \to 1}$

$V_{A \to 1} = -E\Delta r$

or $V_{A \to 1} = -\dfrac{Kq}{r^2}(r_1 - r_A)$

or $V_{A \to 1} = -\dfrac{Kq}{(\sqrt{r_A r_1})^2}(r_1 - r_A)$

or $V_{A \to 1} = \dfrac{Kq}{r_A r_1}(r_A - r_1)$

$V_{A \to 1} = Kq\left(\dfrac{r_A}{r_A r_1} - \dfrac{r_1}{r_A r_1}\right)$

$V_{A \to 1} = Kq\left(\dfrac{1}{r_1} - \dfrac{1}{r_A}\right)$

Similarly

$V_{1 \to 2} = Kq\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$

$V_{2 \to 3} = Kq\left(\dfrac{1}{r_3} - \dfrac{1}{r_2}\right)$

$\vdots \qquad \vdots \qquad \vdots$

& $V_{n \to B} = Kq\left(\dfrac{1}{r_B} - \dfrac{1}{r_n}\right)$

Now from equation (i)

$V_{A \to B} = V_{A \to 1} + V_{1 \to 2} + V_{2 \to 3} + \ldots\ldots\ldots\ldots + V_{n \to B}$

$V_{A \to B} = Kq\left(\dfrac{1}{r_1} - \dfrac{1}{r_A}\right) + Kq\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right) + Kq\left(\dfrac{1}{r_3} - \dfrac{1}{r_2}\right) + \ldots\ldots\ldots\ldots\ldots\ldots + Kq\left(\dfrac{1}{r_B} - \dfrac{1}{r_n}\right)$

$V_{A \to B} = Kq\left(\dfrac{1}{r_1} - \dfrac{1}{r_A} + \dfrac{1}{r_2} - \dfrac{1}{r_1} + \dfrac{1}{r_3} - \dfrac{1}{r_2} + \ldots\ldots\ldots\ldots\ldots\ldots + \dfrac{1}{r_B} - \dfrac{1}{r_n}\right)$

$V_{A \to B} = Kq\left(\dfrac{1}{r_B} - \dfrac{1}{r_A}\right)$

or $\boxed{V_{A \to B} = \Delta V = V_{B \to A} = \dfrac{q}{4\pi\varepsilon_o}\ \dfrac{1}{r_B} - \dfrac{1}{r_A}}$

ABSOLUTE POTENTIAL:

The absolute potential at a point is defined as the amount of work done per unit charge to bring it to that point from infinity against the field.

Thus $V_A = 0$ & $r_A = \infty$ Therefore the absolute potential at B is

$$V_B = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - \frac{1}{r_\infty}\right)$$

$$V_B = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - 0\right) \qquad [R/\infty = 0]$$

We can say that absolute potential at any point at a distance 'r' is

$$\boxed{V = \frac{q}{4\pi\varepsilon_o} \times 1}$$

EQUILIBRIUM POTENTIAL SURFACES:

A surface around a point charge which such a set of points that have same potential.

Consider an isolated point charge. The radial lines are perpendicular to the equipotential lines.

$$V_B - V_A = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Since $r_A = r_B$

$$V_B = V_A$$

Therefore points B & A have the same potential.

ELECTRON VOLT:

Electron volt is the unit of energy used in atomic physics.

When a charge q is moved from a point at higher potential to a point at lower potential, the electric potential energy is reduced by $q\Delta V$. This decrease in P.E. appears as K.E.

i.e. $q\Delta V = K.E.$

& $q\Delta V = \frac{1}{2}mv^2$

From this energy equation

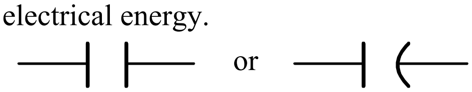
1 electron volt = 1 quantum of charge x 1 volt

$1\ eV = (1.6\times10^{-19}C)(1\ volt)$

or $\boxed{1\ eV = 1.6\times10^{19}\ J}$

CAPACITORS:

Capacitor is a system consists of two conductors of any shape (called plates) separated by each other by an insulating medium called dielectric carrying equal & opposite charge. A capacitor has the ability to store electric charge or electrical energy.



Symbol for capacitor

CAPACITANCE (C):

The capacity of a capacitor to store electric charge is called 'capacitance'. It is dependant on the geometry of plates and the medium used.

Mathematically capacitance is the ratio of charge on one plate (q) & the potential difference (V) between the plates.

$$C = \frac{q}{V}$$

S.I unit of capacitance is Farad (F). One farad (F) is defined as the capacitance of a capacitor if a charge of 1C raises the potential difference between the plates by 1V.

PARALLEL PLATES CAPACITORS:

A parallel plate capacitor consists of (i) two metal plates placed parallel to each other (ii) an insulating medium, called dielectric.

When a battery is connected to the capacitor, a movement of electron from the negative terminal of the battery on to the plate A starts. At the same rate, electrons also flow from the plate B towards the positive terminal of battery. Positive & negative charges thus appears on the plates. As the charges accumulate, the p.d. between the plates increases. The charging current falls to zero when the potential difference between equal to the battery voltage V.

Experimentally:

$$Q \text{ (Charge)} \propto V \text{ (Voltage)}$$

or $$Q = CV$$

Where 'C' is the constant of proportionality and called capacitance of capacitor.

Hence

$$\boxed{C = \frac{Q}{V}}$$

An electric field exists between the plates

& $$E = \frac{V}{d} \quad \rightarrow \text{(i)} \qquad [V = Ed \qquad d = \text{Distance between plates}]$$

But electric field intensity between two oppositely charged plates is

$$E = \frac{\sigma}{\varepsilon_o} \qquad \text{[From Gauss' law]}$$

Where 'σ' is the charge density

i.e. $$\sigma = \frac{Q}{A}$$

or $$E = \frac{Q}{\varepsilon_o A}$$

Putting in equation (i)

$$V = \frac{Q}{\varepsilon_o A} \cdot d$$

Putting V in $C = \frac{Q}{V}$

$$C = Q \quad \frac{Q.d}{\varepsilon_o A}$$

or $$C = Q \times \frac{\varepsilon_o A}{Q.d}$$

or $$\boxed{C = \frac{\varepsilon_o A}{d}} \rightarrow \text{(ii)}$$

CAPACITOR WITH DIELECTRIC:

Now if a dielectric of relative permittivity ε_r is kept between the plates them

$$C' = \frac{Q}{V'} \rightarrow \text{(iii)}$$

& $$E' = \frac{\sigma}{\varepsilon_o \varepsilon_r} = \frac{Q}{\varepsilon_o \varepsilon_r}$$

& $$V' = E'd$$

or $$V' = \frac{Qd}{\varepsilon_r \varepsilon_o A}$$

Putting in equation (iii)

$$C' = Q \quad \frac{Qd}{\varepsilon_r \varepsilon_o A}$$

$$C' = Q \times \frac{\varepsilon_r \varepsilon_o A}{Qd}$$

$$\boxed{C' = \frac{\varepsilon_r \varepsilon_o A}{d}} \rightarrow \text{(iv)}$$

Now comparing equation (ii) & (iii)

$$\frac{C'}{C} = \frac{\varepsilon_r \varepsilon_o A / d}{\varepsilon_o A / d}$$

$$\frac{C'}{C} = \varepsilon_r$$

$$\boxed{C' = \varepsilon_r C}$$

Therefore a capacitor having dielectric between its plates has capacitance ε_r time of capacitance of capacitor without dielectric.

COMBINATION OF CAPACITORS:

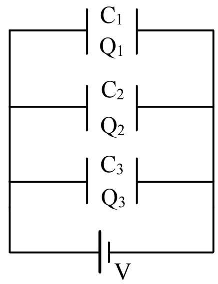
Capacitors can be connected in a circuit either in series or in parallel.

1) CAPACITORS IN PARALLEL:

Capacitors are said to be connected in parallel if they are joined across the same points.

If we consider three capacitors of capacitance C_1, C_2 & C_3 connected in parallel with a battery of potential V. The charge on each capacitor is different & proportional to their capacitances but the potential difference across each of them is same.

Now if the applied potential difference is V them from the figure.



Change on first capacitor $Q_1 = C_1V$

Change on Second capacitor $Q_2 = C_2V$

Change on Third capacitor $Q_3 = C_3V$

If we concide an equivalent capacitor of capacitance C_e then

$$Q = C_eV$$

Where,

$$Q = \text{total charge or } Q = Q_1 + Q_2 + Q_3$$

$$Q = Q_1 + Q_2 + Q_3$$

or $$C_eV = C_1V + C_2V + C_3V$$

or $$C_eV = V\,(C_1 + C_2 + C_3)$$

or $$\boxed{C_e = C_1 + C_2 + C_3}$$

or for 'n' capacitors in series

$$\boxed{C_e = C_1 + C_2 + C_3 + \ldots\ldots\ldots + C_n}$$

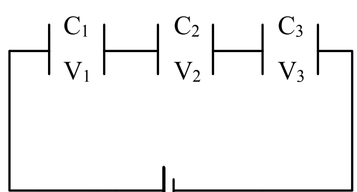
Therefore we can conclude that the equivalent or net capacitance of capacitors connected in parallel is simply equal to the sum of capacitances of all capacitors.

or $$C_e = \sum_{i=1}^{n} C_i$$

2) CAPACITORS IN SERIES:

If many capacitors are connected in a circuit end-to-end then is called series combination of capacitors.

Consider three capacitors of capacitances C_1, C_2 & C_3 are joined in series then the charge stored on then is same but p.d. across each capacitor is different according to their capacitances. As shown in the figure.



Now let 'Q' be the charge on each capacitor

Then p.d. on first capacitor $= V_1 = \frac{Q}{C_1}$

p.d. on first capacitor $= V_2 = \frac{Q}{C_2}$

p.d. on first capacitor $= V_3 = \frac{Q}{C_3}$

Now if we consider an equivalent capacitor of capacitance 'C_e' and the applied voltage is V then

$$V = V_1 + V_2 + V_3$$

or
$$\frac{Q}{C_e} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

or
$$\frac{Q}{C_e} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or
$$\boxed{\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

or for 'n' capacitors

$$\frac{1}{C_e} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots + \frac{1}{C_n}\right)$$

or
$$C_e = \sum_{i=1}^{n} C_i$$

Also
$$\boxed{C_e = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots + \frac{1}{C_n}\right)^{-1}}$$

ENERGY STORED IN CAPACITOR:

If plates of a charged capacitor are connected with a conductor then they loose their charge (discharging). As a result of this discharging spark & heat energy produced which proves that a charged capacitor has energy.

Now if Q charge is stored on the plate by doing work against field then it appears as the stored energy.

i.e. Work done in charging = charge x average potential difference

or
$$\text{Energy} = Q \times \left(\frac{V+0}{2}\right)$$

$$\text{Energy} = \frac{1}{2} QV \quad \rightarrow \text{(i)}$$

$$\text{Energy} = \frac{1}{2} CV^2 \quad \rightarrow \text{(ii)} \qquad [Q = CV \quad : C = Q/C]$$

or
$$\text{Energy} = \frac{1}{2} C\left(\frac{Q}{C}\right)^2$$

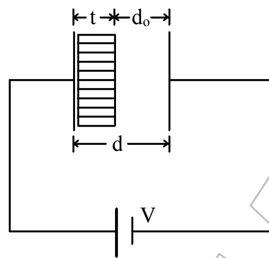
$$\text{Energy} = \frac{1}{2}\frac{Q^2}{C} \quad \rightarrow \text{(iii)}$$

$$\boxed{E = \frac{1}{2}QV + \frac{1}{2}CV^2 + \frac{1}{2}\frac{Q^2}{C}}$$

COMPOUND CAPACITOR:

A compound capacitor is such a capacitor in which the space between two plates of capacitor has two dielectric medium between its plates.

Consider the figure below:



Let

d = distance between plates
t = thickness of dielectric
$d_o = d - t$ = thickness of air

In fact the system consists of two capacitor in series.

If C = capacitance for the capacitor with air
C' = capacitance for the capacitor with dielectric

Then Net capacitance C_d

$$\frac{1}{C_d} = \frac{1}{C} + \frac{1}{C'}$$

or
$$\frac{1}{C_d} = \frac{C + C'}{CC'}$$

or
$$C_d = \frac{CC'}{C + C'}$$

But
$$C = \frac{\varepsilon_o A}{d_o} \qquad \& \qquad C' = \frac{\varepsilon_o \varepsilon_r A}{t}$$

$$C_d = \frac{\dfrac{\varepsilon_o A}{d_o} \times \dfrac{\varepsilon_o \varepsilon_r A}{t}}{\dfrac{\varepsilon_o A}{d_o} + \dfrac{\varepsilon_o \varepsilon_r A}{t}}$$

or
$$C_d = \frac{\dfrac{\varepsilon_o A}{d_o} \times \dfrac{\varepsilon_o \varepsilon_r A}{t}}{\dfrac{\varepsilon_o A t + \varepsilon_o \varepsilon_r A d_o}{d_o t}}$$

or
$$C_d = \frac{\dfrac{\varepsilon_o A}{d_o} \times \dfrac{\varepsilon_o \varepsilon_r A}{t}}{\varepsilon_o A \left(\dfrac{t + \varepsilon_r d_o}{d_o t}\right)}$$

or $$C_d = \frac{\varepsilon_o \varepsilon_r A}{t + \varepsilon_r d_o}$$

or $$C_d = \frac{\varepsilon_o \varepsilon_r A}{\varepsilon_r\left(\frac{t}{\varepsilon_r} + d_o\right)}$$

or $$\boxed{C_d = \frac{\varepsilon_o A}{\frac{t}{\varepsilon_r} + (d - t)}}$$

XII PHYSICS

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CHAPTER # **13**

CURRENT ELECTRICITY

ELECTRIC CURRENT:

"Electric current is defined as the quantity of charge (electrons) which passes through any section of a conductor in unit time."

Mathematically;

If (Q) charges pass through the cross-section of a conductor in time (t) then current (I) will be;

$$\boxed{I = \frac{Q}{t}}$$

Unit:

$$I = \frac{Q\ (c)}{t\ (s)} = cs^{-1}$$

$$\boxed{I = \text{Ampere (A)}}$$

S.I unit of electric current is Ampere (A), such that

$$1A = \frac{1c}{1s}$$

AMPERE (A):

1 ampere is defined as the quantity of electric current when a charge of 1 coulomb pass through a cross-section of a conductor in 1 second.

DIRECTION OF ELECTRIC CURRENT

The direction of electric current can be expressed equally in the following two ways.

1) ELECTRONIC CURRENT:

"The electric current in fact, is caused by flow (drifting) of free electrons from lower potential (-ve terminal) to the higher potential (+ve terminal). This direction of current is known as electronic current."

2) CONVENTIONAL CURRENT:

"Previously it was believed that the electric current is the flow positive charge through a circuit from a higher potential (+ve terminal) to a lower potential (-ve terminal). The current caused by this flow of equivalent positive charge is called conventional current."

OHM'S LAW:

This law was put forward by George Simon Ohm in 1826, regarding the relation between the potential difference across a conductor and the current following through it.

Statement (1):

"The potential difference (V) across the ends of a conductor is directly proportional to the current (I) passing through it provided that the physical state (density, temperature etc) of conductor remains the same."

Mathematically;

Potential difference (P.d) or voltage α current

or $V \propto I$

or $\boxed{V = IR}$

Where 'R' is the constant of proportionality and it is known as the "RESISTANCE" of the circuit.

RESISTANCE:

"Resistance is defined as the opposition or obstacles faced by the flow of electric current through a conductor."

Unit:

$$V = IR$$

$$R = \frac{V}{I}$$

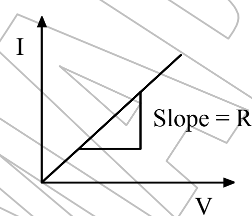
$$R = \frac{V\ (\text{volts})}{I\ (\text{ampere})}$$

$$R = VA^{-1}$$

$$\boxed{R = \text{Ohm}\ (\Omega)}$$

S.I unit of resistance is ohm (Ω) such that

$$1\ \text{ohm}\ (\Omega) = \frac{1\ \text{volt (V)}}{1\ \text{amp. (A)}}$$



Ohm (Ω):

"1 ohm (Ω) is defined as the resistance of a circuit when a current of 1 Ampere (A) passes through a conductor under a potential difference of 1 volt (V)."

Ohm's law is valid for certain conductors & not valid for certain conductors as well. Such conductors are called Ohmic-conductors & non-Ohmic-conductors respectively. For a non-Ohmic conductor, the graph between voltage & current is not a straight line.

Statement (2):

Ohm's law can also be stated as following:

"The current through a conductor is directly proportional to the potential difference or voltage (V) across the ends of the conductor."

Mathematically;

$$I \propto V$$

or $\boxed{I = KV}$

Where 'K' is the constant of proportionality and it is known as the conductance of circuit.

CONDUCTANCE (K):

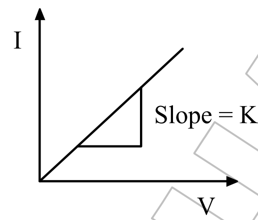
Conductance of the circuit is defined as the "facility" to the flow of electric current through a conductor.

As $I = KV$

or $K = \dfrac{I}{V}$

or $K = \dfrac{1}{V/I}$

or $\boxed{K = \dfrac{1}{R}}$ $[R = V/I]$



The conductance is the opposite of resistance and hence its S.I unit is (Ω^{-1}).

Furthermore;

$1\ (\Omega^{-1}) = 1$ siemens (S)

RESISTIVITY (SPECIFIC RESISTANCE):

The resistance of a conductor is directly proportional to its length and inversely proportional to its area of cross-section.

i.e. $R \ \alpha \ L$

& $R \ \alpha \ 1/A$

Combining these factors

$R \ \alpha \ \dfrac{L}{A}$

or $R = \ell \dfrac{L}{A}$

Where 'ℓ' is the constant of proportionality and it is known as the "resistivity" of the material.

Further;

$\boxed{\ell = \dfrac{RA}{L}}$

"Resistivity may be defined as the resistance of the conductor per unit length per unit area of cross-section of the conductor." In other words resistivity is the resistance of a one meter cube of the material.

Unit:

$$\ell = \frac{RA}{L} = \frac{R\,(\Omega).\,A\,(m^2)}{L\,(m)}$$

$$\boxed{\ell = \Omega.m}$$

S.I unit of Resistivity (ℓ) is ohm-meter (Ω.m). the inverse of resistivity is conductivity (σ).

& $\boxed{\sigma = 1/\ell}$ hence its S.I unit is $(\Omega.m)^{-1}$ or $(S.m^{-1})$.

TEMPERATURE CO-EFFICIENT OF RESISTANCE:

In fact, the resistance of a conductor is caused by the collisions of free electrons with atomic lattice. Therefore, when temperature of a conductor increases, the amplitude vibration for these atoms increases and which increases the probability of collisions of free electrons with them. Consequently this results in an increase of resistance of a conductor.

Experimentally, the change in resistance of a conductor with temperature is nearly linear over a wide range of temperature above and below 0^oC.

Let the resistance of a conductor at $0^oC = R_o$
Let the resistance of a conductor at $t^oC = R_t$

Change in temperature = ΔT
Change resistance = $R_t - R_o = \Delta R$

& Change in resistance per unit resistance = $\frac{R_t - R_o}{R_o}$

or Change in resistance per unit resistance per unit ^oC = $\frac{R_t - R_o}{R_o \Delta T}$

& the change in resistance per unit resistance per unit ^oC is called temperature coefficient of resistance (α)

$$\boxed{\alpha = \frac{R_t - R_o}{R_o \Delta T}} \rightarrow \text{(i)}$$

According to the above expression the resistivity of the conductor is defines as the fractional change in resistance per Kelvin or per degree Celsius.

Unit: S.I unit of 'α' is $^oC^{-1}$ or K^{-1}.

Also $\boxed{R_t = R_o\,(1 + \alpha\,\Delta T)}$

TEMPERATURE COEFFICIENT OF RESISTIVITY:

Since resistance is proportional to the resistivity therefore;

$$\boxed{\alpha = \frac{\ell_t - \ell_o}{\ell_o \Delta T}}$$

or $\boxed{\ell_t = \ell_o\,(1 + \alpha\,\Delta T)}$

COMBINATION OF RESISTORS

Like all other elements or components of an electric circuit, Resistors can also be connected either in (a) series or (b) in parallel.

1) RESISTORS IN SERIES:

Resistors are said to be connected series if there is only one path for the current to flow through them.

In a series connected the potential difference across each resistor is different but the current passing through them is the same.

From the figure (1)

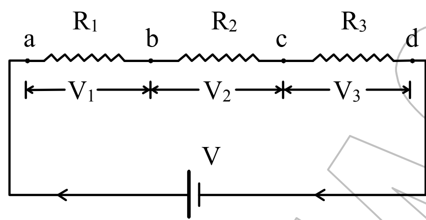


Figure (1)

If the equivalent Resistance of the circuit is (Reconciliation), Potential difference applied is (V) then according to Ohm's law

$V = IR_e \quad \rightarrow \text{(i)}$

Similarly, V_{ab} or $V_1 = IR_1$

V_{bc} or $V_2 = IR_2$

V_{cd} or $V_3 = IR_3$

'I' is same for all resistors.

Since

$V = V_1 + V_2 + V_3$

Putting the values

or $IR_e = IR_1 + IR_2 + IR_3$

or $IR_e = I\,(R_1 + R_2 + R_3)$

$$\boxed{R_e = R_1 + R_2 + R_3}$$

The above expression can be used to determine the equivalent resistance of a circuit containing 'n' number of resistance in series.

i.e. $\boxed{R_e = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots + R_n}$

2) RESISTORS IN PARALLEL:

If resistors are joined across the same two points in such a way that each resistor is provided an alternate path to the current, then this arrangement is called combination of Resistors in parallel.

In parallel combination, the potential difference across each resistor is same but the current flowing through them is different.

From the figure (2)

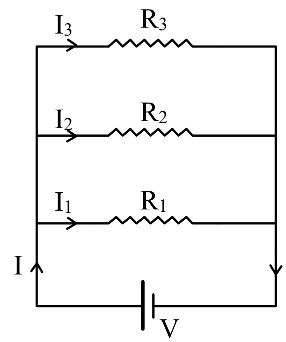


Figure (2)

If the applied voltage is 'V' & the equivalent resistance is 'Reconciliation' then according to Ohm's law

$$I = \frac{V}{R_e} \qquad \rightarrow \text{(i)}$$

Similarly the current through resistors R_1, R_2 & R_3 should be

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

$$I_3 = \frac{V}{R_3}$$

& the total current 'I'

$$I = I_1 + I_2 + I_3$$

Putting the values

$$\frac{V}{R_e} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or

$$\frac{V}{R_e} = V \quad \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or

$$\boxed{\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

For 'n' resistors connected in parallel, the equivalent resistance can be evaluated as following:

$$\boxed{\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots + \frac{1}{R_n}}$$

POWER DISSIPATION IN RESISTORS:

When a charge flows through a conductor, it dissipates energy in the form of heat. The energy dissipated is equal to the potential energy lost by the charge as it moves through the potential difference that exists between the ends of the conductor.

The energy dissipated as heat = Q x V [V = W/Q]

& The power dissipated as heat = $\frac{Q \times V}{t}$ [P = W/t]

or $P = V \times \frac{Q}{t}$ [I = Q/t]

$\boxed{P = VI}$

or P = (IR) I [V = IR]

$\boxed{P = I^2R}$

or $P = V \frac{V}{R}$ [I = V/R]

$\boxed{P = \frac{V^2}{R}}$

"Electrical Power of a device may be defined as the rate at which the electric energy is converted into other forms of energy."

Unit:

As P = VI

P = V (Volts) . I (Amp.)

P = VA

P = Watt (W)

The S.I unit of power is watt (W) such that

1 Watt (W) = 1 Volt (V) x 1 Amp. (A)

Another unit of Power is horse-Power such that

1 horse power (h.p) = 746 Watt (W)

JOULES'S LAW OF ELECTRIC HEATING:

The electrical energy spent to maintain a current (I) for time (t) appeared as heat

Energy dissipated as heat = Power dissipated x time

$$\left.\begin{aligned} \text{or} \quad & H = VI \times t \\ \text{or} \quad & H = I^2R \times t \\ \text{or} \quad & H = \frac{V^2}{R} \times t \end{aligned}\right\} \quad \text{[Joule's law of electric heating]}$$

PRACTICAL UNIT OF ELECTRIC ENERGY:

The practical unit for the measurement of electric energy is "kilo-Watt-hour" (kWh) which is known as one unit. Such that

$$1\ kWh = 1000\ Wh$$
$$= 1000 \times \frac{J}{s} \times 60 \times 60\ s$$
$$= 3600 \times 1000\ J$$

$$\boxed{1\ kWh = 3.6 \times 10^6\ J}$$

ELECTROMOTIVE FORCE (e.m.f):

The electromotive force (e.m.f) of a source is defined as the that work done required to drive a unit charge round a complete electric circuit in which the source is connected (from low to high potential).

Thus, $$\text{e.m.f } (E) = \frac{\text{Energy or Work}}{\text{Charge}}$$

$$\boxed{E = \frac{W}{Q}}$$

TERMINAL POTENTIAL DIFFERENCE:

The resistance offered by a source or e.m.f across the terminals to the passage of current is called "internal resistance" (r). This is due to the resistance of electrodes, terminals & electrolyte of the cell.

When no current is drawn from the source, there is no potential drop across the internal resistance & therefore the potential difference between the terminals of a battery is equal to e.m.f.

i.e. $$V = E$$

or $$E = IR$$

But when circuit is closed, some power is lost as thermal energy in external & internal resistor as well.

$$P_E = P_{Ext} + P_{Int}$$

or $$IE = IV_{ab} + IV_{bc}$$

or $$IE = I^2R + I^2r$$

$$IE = I^2\ (R + r)$$

$$\boxed{E = I(R + r)}$$

Here V(IR) represents the terminal Potential difference.

or $$E = IR + Ir$$

$$\boxed{E = V + Ir}$$

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CHAPTER # **14**

MAGNETISM & ELECTROMAGNETISM

MAGNETIC FIELD DUE TO CURRENT:

In 1819 Oersted discovered that a magnetic field is produced a current carrying wire or conductor which affects nearby magnetic needle. The direction of magnetic field can be determined using 'right hand grip rule' according to which

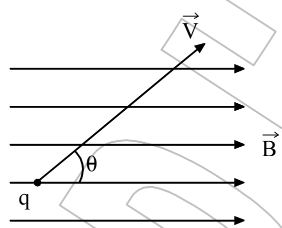
"If the current carrying wire is grasped is right hand is such a way that the thumb shows the direction of current then the curl of fingers will indicate the direction of lines of magnetic field."

Ampere discovered in 1820 that if two current carrying wires are kept parallel then they will attract each other if the current flows in the same direction. However they will repel each other if the direction of current is opposite in both the wires.

FORECE ON A CHARGED PARTICLE IN MAGNETIC FIELD:

A magnetic field is caused when an electric charge moves. If an electric charge is moved through an applied magnetic field then due to the interaction of both these fields, a force is experienced by the charge which depends or proportional to

1) the quantity of charge
2) the velocity of charge
3) the strength of magnetic field
4) Sinθ (i.e. the force is maximum perpendicular to the field).



If an electric charge 'q' moves in a magnetic field of strength 'B' with velocity 'V' making angle 'θ' with the field then symbolically the force 'F' should be,

$$F \propto VBq\sin\theta$$

The unit of 'B' in MKS system is such that the constant of proportionality becomes unity.

$$\boxed{F = qVB\sin\theta}$$

Unit of 'B'

Since $F = qVB\sin\theta$

$$B = \frac{F}{qV\sin\theta}$$

Hence the S.I unit of 'B' is Tesla (T).

If q 1C, V = m/s, F = 1N & $\theta = 90^o$

Then $$B = \frac{1N}{1C.1m/s.\ \sin 90^o} = \frac{N}{C/s.\ m.1} = \frac{N}{A.m}$$

or $$\boxed{1\ N.A^{-1}.m^{-1} = 1\ T\ (\text{Tesla})}$$

Definition of B:

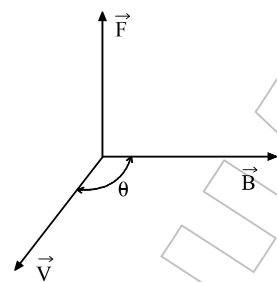
A unit magnetic field of indication is said to exist at a point where a charge of one coulomb moving at right angle to the field with a velocity of one meter per second experiences a force of one Newton.

The unit $N.A^{-1}.m^{-1}$ may also be called Webers per square meter ($Wb.m^{-2}$) where webers is the unit of Magnetic flux.

$$1\ N.A^{-1}.m^{-1} = 1\ T\ (\text{Tesla}) = 1\ Web.m^{-2}$$

In vector form the above formula may be expressed as

$$\boxed{F = q\ (V \times B)}$$



FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD:

If a current carrying conductor is placed in a magnetic field then it experiences a force. This force can be calculated as following:

Let the length of conductor = L
Area of crossection of conductor = A
Volume of conductor = AL
If no. of electrons per unit volume is 'n'
Then total no. of electrons = nAL

If 'e' is the charge of one electron then
Total charge = q= enAL → (i)

Since i9t has been observed that a moving charge experiences a force in a magnetic field due to interaction of fields. An electric current through a conductor is caused by the drifting of free electrons through it which causes the magnetic field around it and when this conductor is brought inside a magnetic field, then due to the interaction of both fields, the conductor experiences a force

$$F = q\ (V \times B)$$

Putting q = neAL from equation (i)

$$F = neAL\ (V \times B)$$

If a is a unit vector in the direction of V then

$$F = neAL\ (aV \times B)$$

or $$F = neAV\ (aL \times B)$$

If displacement vector L = aL then

$$F = nAeV\,(L \times B)$$

If charge 'q' takes time 't' to cross the length 'L' of conductor then drift velocity 'V' should be

$$V = \frac{L}{t}$$

$$F = \frac{nAeL}{t}\,(L \times B) \qquad \rightarrow \text{(ii)}$$

As $I = q/t$ but $q = neAL$

$$I = \frac{nAeL}{t}$$

Putting $I = nAel/t$ from equation (ii)

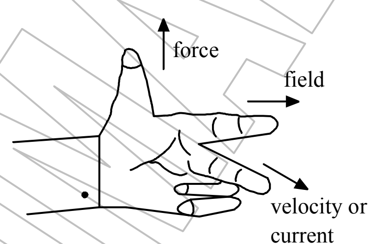
$$F = I\,(L \times B)$$

or $$\boxed{F = ILB\sin\theta}$$

For Maximum force $\theta = 90^o$

or $$F_{\perp} = ILB \text{ (Max. Force)}$$

The force produced is perpendicular to the plane of both L & B & its direction can be determined using right hand rule Fleming's left hand rule.



From the above expression the magnetic field of induction may be defined as the force exerted on a conductor of unit length carrying one ampere current, plane at right angle to the field.

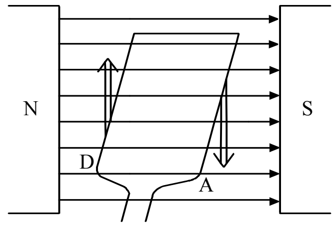
Where 1 Tesla (T) may be defined as the strength of a magnetic field in which a force of one Newton acts on one meter length of a conductor, placed perpendicularly, carrying a current of one ampere.

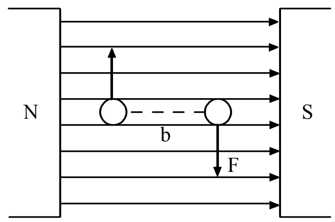
FORCE ON A CURRENT CARRYING COIL IN A MAGNETIC FIELD:

When a current carrying coil is placed in a magnetic field in such a way that its plane is parallel to the field then a torque is produced which tends to rotate the coil in the field.

The torque, thus produced may be derived as following:

Let a rectangular coil (ABCD) be placed in a magnetic field of strength 'B'. Let the plane of coil be parallel with the field and the coil is free to move about an axis XX′.





Let the length of coil = AB = CD = L
Let the breath of coil = BC = DA = b
Let current through the coil = T

When current 'I' is passed through the coil, a force is experienced on the perpendicularly placed conductors AB & CD. The magnitude of the force is F = ILB but no force acts on sides BC & DA as they are parallel to the field.

According to right hand rule the force on side AB is directed upward and the force on BC is directed downward. Hence a couple acts on the coil. Due to this couple the coil rotates.

Mathematically;

Torque due to couple = Magnitude of either force x Perpendicular distance between the forces

$\tau = F_{\perp} b$

or $\tau = ILB.b$ $[F_{\perp} = ILB]$

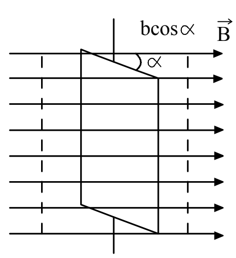
or $\tau = IB\,(Lb)$

or $\tau = BIA$ $[Lb = \text{Area} = A]$

If, during the rotation, the coil makes angle 'α' with the field the perpendicular distance between both force becomes 'bcosα' therefore

$\tau = BIA\cos\alpha$

If there are 'N' no. of turns in coil then



$\boxed{\tau = BINA\cos\alpha}$

MAGNETIC FLUX (Ø_m):

Magnetic flux is a scalar property of a vector field (B). It is defined as the scalar products of Magentic field 'B' and small element of Area 'A'.

i.e. $\boxed{\text{Ø}_m = B.\Delta A}$

Where the direction of area is indicated by an outward drawn the normal on the surface.

If 'θ' is the angle between B & A then

$$\boxed{\text{Ø}_m = B\Delta A\cos\theta}$$

When $\theta = 0^o$

$$\text{Ø}_m = B\Delta A\cos 0^o$$

$$\text{Ø}_m = B\Delta A(1) \qquad \text{(Maximum)}$$

When $\theta = 90^o$

$$\text{Ø}_m = B\Delta S\cos 90^o$$

$$\text{Ø}_m = 0$$

Unit:

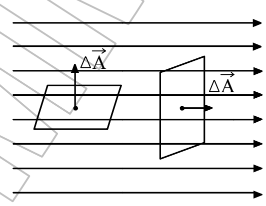
S.I unit of Magentic flux is $= \dfrac{N}{A.m} . m^2 = N.m.A^{-1}$

Here $\boxed{1\ N.m.A^{-1} = 1 \text{ Weber (W)}}$

Thus S.I unit of magnetic flux is Webers (Wb).

MAGNETIC FLUX DENSITY (B):

"Magnetic flux passing through unit area surface perpendicularly is called Magnetic Flux Density or Magnetic Induction."



i.e. $\boxed{B = \dfrac{\text{Ø}_m}{\Delta A}}$

Unit: S.I unit of B is ($Wb.m^{-2}$).

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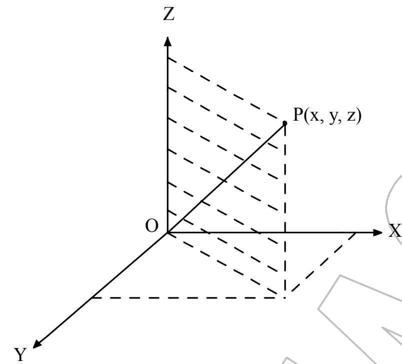
CHAPTER # 17

MODERN PHYSICS

FRAME OF REFERENCE:

A frame of reference is infact a set of coordinated axis which consists of a reference point called origin upon which three mutually orthogonal lines are fixed. Position of a body can be represented by three cartesian coordinates. This rectangular coordinate system can represent the position, displacement and velocity of a body. An observer has a fixed location with respect to the coordinate axis system. The observer and this system of coordinate axis are collectively called "frame of reference".



INERTIAL FRAME OF REFERENCE:

Inertial frames of reference are also called Newtonian frames or Galilean frames. These are such frames of reference which remain at rest or move with uniform velocity. In inertial frames of reference all Newton's laws of motion hold good and these frames of reference have zero acceleration (linear or angular both).

FRAME OF REFERENCE IN UNIFORM RELATIVE MOTION:
OR
(GALILEAN TRANSFORMATION):

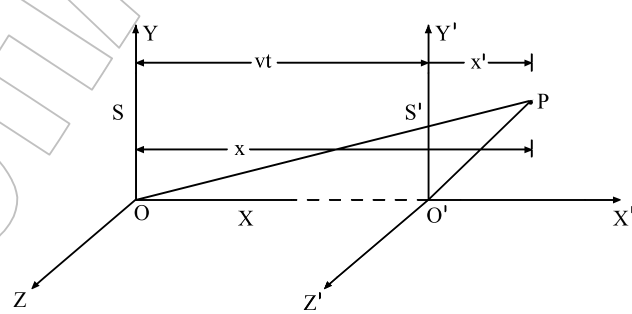
The conversion of co-ordinates of a particle from an inertial frame to another inertial frame of reference is called "Galilean transformation". Such sets of equations are called transformation equations. They enable us to pass from space coordinates and times of events in one frame of reference to coordinates and times in another frame when both are in relative motion.

HYPOTHESIS OF GALILEAN:

"The basic laws of Physics remain unchanged in form in two frames of reference connected by Galilean transformation provided the two frames move with constant.

EXPLANATION:

If Newton's second law of motion ($F = ma$) is valid in an inertial frame of reference $S(x, y, z)$ at rest at $t = 0$ then it should also hold good in another $S'(x', y', z')$ moving at constant velocity with respect to frame S. Let the observer 'O' is in the frame 'S' and 'O' in the frame 'S''.



TRANSFORMATION OF SPACE:

Let the origins of frames 'S' & 'S′' are coincident when t = 0 and then frame 'S′' move with relative uniform velocity 'V'. After time 't' measurement are taken at point 'P' designated as (x, y, z) with respect to frame 'S' and (x′, y′, z′) with respect to frame 'S′'.

From the above figure

$x = x' + Vt \quad \rightarrow \text{(i)}$

$y = y' \qquad \& \qquad t = t'$

$z = z'$

This is the Galilean transformation of space-time coordinates of a point P. Here time is absolute.

VELOCITY TRANSFORMATION:

Dividing equation (i) by time

$$\frac{x}{t} = \frac{x'}{t} + \frac{Vt}{t}$$

$$\boxed{V = V' + \upsilon} \quad \rightarrow \text{(ii)}$$

Where,

V = Velocity of Particle 'P' measured in S.
V′ = Velocity of Particle 'P' measured in S′.
υ = Uniform velocity of frame S′.

TRANSFORMATION OF ACCELERATION:

If V_1 & V_2 are initial & final velocities of particle 'P' measured in S & V_1' & V_2' are initial & final velocities of particle 'P' measured in 'S′' then acceleration 'a' measured in S will be

$$a = \frac{V_2 - V_1}{t} \qquad \& \qquad \text{Similarly} \qquad a' = \frac{V_2' - V_1'}{t} \quad \rightarrow \text{(iii)}$$

As $\quad V = V' + \upsilon \qquad$ (from equation ii)

$V_1 = V_1' + \upsilon$

or $\quad V_1' = V_1 - \upsilon$

& $\quad V_2 = V_2' + \upsilon$

or $\quad V_2' = V_2 - \upsilon$

Putting in equation (iii)

$$a' = \frac{(V_2 - \upsilon) - (V_1 - \upsilon)}{t}$$

$$a' = \frac{V_2 - \upsilon - V_1 + \upsilon}{t}$$

$$a' = \underline{V_2 - V_1}$$

t

$\boxed{a' = a}$

Thus the acceleration measured in moving frame is just equal to measured in Stationary frame.

<u>TRANSFORMATION OF FORCE:</u>

$F = ma$ & $F' = ma'$

But $a' = a$

$\boxed{F = F' = ma}$

It means both the observers would agree on the magnitude and direction of force regardless the relative velocities of inertial frames of reference.

<u>SPECIAL THEORY OF RELATIVITY:</u>

The Special theory of Relativity was proposed by Einstein in 1905 which was developed specially for inertial frames. This theory states that all physical concepts are meaningful only in relation to an observer in inertial frames.

This theory is based on the following two postulates.

1) <u>PRINCIPLE OF RELATIVITY:</u>

All physical laws may be expressed in the same equations for all frames of reference moving at constant velocity with respect to each other.

2) <u>CONSTANCY OF SPEED OF LIGHT:</u>

The speed of light in free space is constant for all observers, regardless their state of rest or motion.

<u>CONSEQUENCES OF SPECIAL THEORY OF RELATIVITY:</u>

1) State of absolute motion or absolute rest is meaningless. All motions are relative.
2) Assumption of ether is not needed any more. All physical phenomenon's can be expressed by relative motion.
3) Space & time, both are relative.
4) Events take place in "four-dimensional continuum" called "Space time continuum". Infact the world has four dimensions. Three coordinates x, y & z and fourth coordinate is time.
5) The mass of body moving with velocity 'V' relative to the observer is larger than its mass at rest. i.e.

$$\boxed{m = \frac{m_o}{\sqrt{1 - V^2/c^2}}} \quad \rightarrow \text{(i)}$$

Where,

m = Relative mass
m_o = Mass at rest
V = Velocity
c = Speed of light

6) If a body moves with velocity 'V' with respect to an observer then its length appears to be shorter than it is observed at rest. i.e.

$$\boxed{l = l_o \sqrt{1 - V^2/c^2}}$$

Where,

l = Relative length

l_o = Length at rest

7) Time interval or duration of an event increases with respect to a moving observer as compared to the time duration of the same event at the state of rest observer. i.e.

$$\boxed{t = \frac{t_o}{\sqrt{1 - V^2/c^2}}}$$

Where,

t = Relative time

t_o = Time at rest

8) Mass & energy are interconvertable according to following relation called Mass-Energy equation.

$$\boxed{E = mc^2}$$

Where,

E = Energy

m = Mass

c = Speed of light

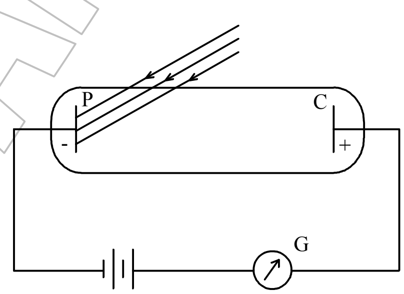
9) Speed of any object can not exceed speed of light because when $V \rightarrow C \Rightarrow V^2/c^2 = 1$ & $\sqrt{1 - V^2/c^2} = 0$ or $m = \infty$ (from equation i) & infinite mass does not exist therefore it can be said that speed of light is the limit for speed of any object in the universe.

PHOTOELECTRIC EFFECT:

The process of photoelectric effect was discovered by R.H. Hertz in 1887. He found that when ultraviolet rays fells on a spark gap, the sparks passed more easily. Infact when high frequency radiations like ultraviolet rays, x-rays, y-rays etc, fall on a metal surface, electrons are emitted. These electrons are called photo electrons and the phenomenon is known as "Photoelectric Effect".

HALLAWACH'S EXPERIMENT:

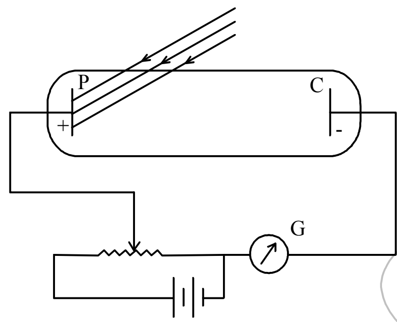
The phenomenon was demonstrated by a scientist named Hallawach. In the experiment, an evacuated quartz tube, containing two electrodes, is connected to a battery through a galvanometer. Light from a source is allowed to fall on plate P (Zinc plate) which is given –ve potential.



The galvanometer indicates deflection (i.e. current flows). This shows that the other plates C (called collector plate) gets electron from the plate P due to irradiation. When light is stopped, then no current flows. This clearly demonstrates that radiations eject electrons from the metal surface.

MAXIMUM KINETIC ENERGY OF PHOTOELECTRONS:

To estimate the maximum kinetic energy of photo electrons, Lannard did an experiment in which he connected and evacuated quartz tube containing two electrodes to an external circuit as shown in the figure below.



The plate P is given +ve & the other plates C (collector) is given a –ve potential using a battery with potential divider.

When radiation fall on plate P, photoelectrons are emitted and reach the plate C despite its –ve potential because these photoelectrons have sufficient kinetic energy to reach plate C. Due to this motion of photoelectrons from plate 'P' to 'C' the galvanometer indicates a current. In other to determine the maximum kinetic energy of photoelectrons, the plate C is made more & more –ve with respect to plate P. Therefore the –ve potential at which the ejection of photoelectrons stops (i.e. current in the galvanometer is zero) is called "Retarding or Stopping Potential" denoted by (V_o). The energy equivalent, then corresponds to Maximum kinetic energy of photoelectrons.

Therefore

$$(K.E)_{max} = V_o e$$

or

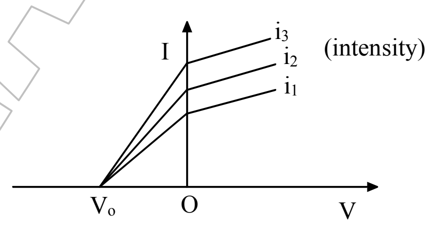
$$\boxed{\frac{1}{2} m\upsilon_o^2 = V_o e}$$

Where,

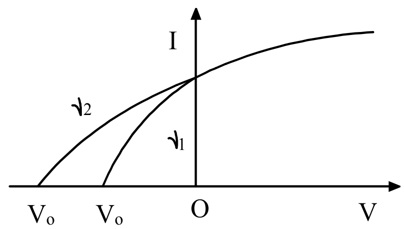
V_o = Stopping potential
e = Charge of one electron
υ = Maximum velocity of electron

EXPERIMENTAL RESULT:

During the phenomenon of photoelectric effect, light of different frequencies and amplitude (intensity) was used, then two types of experimental results were obtained.



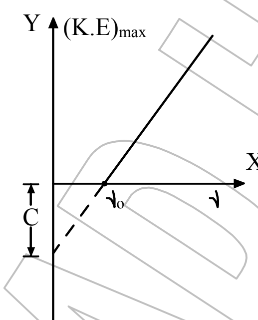
When different radiations having same frequency and different amplitude (intensity) were used then the graph between potential & photoelectric current shows that stopping potential (V_o) is independent of the intensity of source of light.



Whereas when these curves are plotted for same intensity but different frequencies then it was concluded that saturation current depends on the intensity of radiation whereas stopping potential is depended on the frequency of light and it increases with increase of frequency.

THE PHOTOELECTRIC EQUATION:

If we draw a graph between maximum kinetic energy of photoelectrons and frequency of radiation used, then a straight line is obtained which cuts the X-axis at a particular point 'υ_o' called threshold frequency. Threshold frequency is defined as the minimum frequency of radiation required to eject photoelectrons from the surface of metal without giving them additional energy. The value of threshold frequency is dependant on the nature of metal.



The standard equation of straight line is

$$\boxed{y = mx + c}$$

Where,

m = Slope of line
c = Y-intercept

Therefore from the above graph.

$$(K.E)_{max} = A\upsilon - B \qquad \rightarrow (A)$$

or $$V_o e = A\upsilon - B \qquad \rightarrow (B)$$

Equation (A) or (B) represent "Photoelectric Equation".

EXPERIMENTAL OBSERVATION:

1) The photoelectric current or no. of photoelectrons ejected depends on the intensity (amplitude) of radiations used but it is independent of the frequency of radiations.
2) The maximum kinetic energy of photoelectrons depends on the frequency of radiations used above threshold frequency (υ_o).
3) The emission of photoelectrons takes place almost instantaneously ($<10^{-9}$s).

FAILURE OF CLASSICAL PHYSICS:

The phenomenon of photoelectric could not be explained using classical electromagnetic theory of light.

1) The velocity (kinetic energy) of photoelectrons should depend on the amplitude or intensity of radiation but it depends on the frequency not on the intensity.
2) It would have not been possible for photoelectrons to emit out until and unless they acquire sufficient energy to escape. But spontaneous emission of photoelectrons is observed.
3) These should not be threshold frequency because photoelectrons may absorb enough amount of energy from the incident radiations to escape out at any applied frequency.

PROBLEMS:

1) How electrons are emitted spontaneously?
2) How can radiation (wave) emit electrons (particles)?
3) Why a radiation having frequency less than threshold frequency can not eject electrons no mater how intense it is?
4) What is the physical interpretation of constants A & B in photoelectric equation?

EINSTEIN'S EXPLANATION:

Einstein explained the phenomenon of photoelectric effect in 1905 using Planck's quantum theory. According to which energy is absorbed or emitted in the form of packets case of radiation energy. The energy of each quantum is proportional to the frequency of radiation.

i.e. $\boxed{E = h\upsilon}$

Where,

$h = 6.625\text{x}10^{-34}$ J.s (Plank's Constant)

It was assumed by Einstein that when photons fall on the metal surface, following things happen.

1) The collisions between photons & photoelectrons are treated as elastic i.e. no loss of energy.
2) When photon-electron collision takes place, the photon transfers all its energy or none of its energy to the electron.
3) An electron can not absorb more than one photon.

WORK FUNCTION (Ø_o):

The minimum amount of energy or work done required to eject an electron overcoming the attractive forces acting on the electron is called "work function (Ø_o)". An electron is dislodged when energy of photon is larger than work function. Electron emits in this way spontaneously.

The absorbed energy of photon is utilized in two ways.

1) doing work to eject electron out of the metal.
2) giving electron some kinetic energy.

i.e. Quantum energy of incident = work required to eject electron + K.E of photoelectron

or $h\upsilon = \text{Ø}_o + (K.E)_{max}$

or in other way

$\text{Ø}_o = h\upsilon_o$

Where υ_o = threshold frequency

We can say

$h\upsilon = \text{Ø}_o + (K.E)_{max}$ → (i) or $h\upsilon = h\upsilon_o + (K.E)_{max}$ → (ii)

or $h\upsilon = h\upsilon_o + V_oe$ → (iii)

or $V_oe = h\upsilon - Ø_o$ → (iv)

CONCLUSIONS:

On the basis of Einstein's explanation, we can conclude that

1) The photoelectric effect is a particle interaction and not a wave-particle interaction according to quantum nature of light.
2) Since work function ($Ø_o = h\upsilon_o$) corresponds to threshold frequency of radiation and not the intensity of radiation therefore it is concluded that emission of photoelectrons is depended on the frequency of light and not the intensity.
3) A Comparison of equations (B) & (iv) reveals that constant A is infact plank's constant, whereas constant B refers to the work function ($Ø_o$) of the metal.